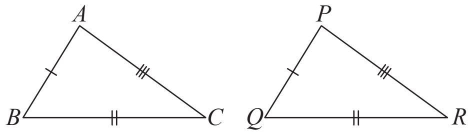
Congruency and Similarity

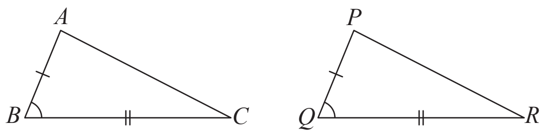
1. Test for Congruency

(a) The 3 corresponding sides of both triangles are equal. (SSS Property)



$AB = PQ$
$BC = QR$
$AC = PR$
$\therefore \Delta ABC \equiv \Delta PQR$ (SSS)

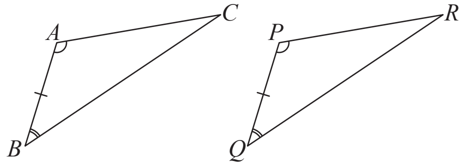
(b) The 2 corresponding sides and the included angle of both triangles are equal. (SAS Property)



$AB = PQ$
$\angle ABC = \angle PQR$
$BC = QR$
$\therefore \Delta ABC \equiv \Delta PQR$ (SAS)

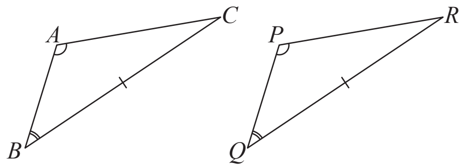
(c) The 2 corresponding angles and a corresponding side of both triangles are equal.

(i) (ASA Property)



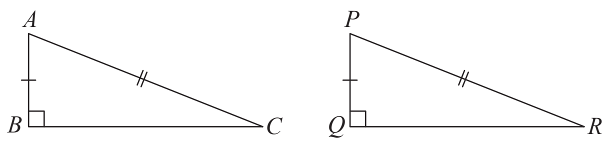
$\angle BAC = \angle QPR$
$AB = PQ$
$\angle ABC = \angle PQR$
$\therefore \Delta ABC \equiv \Delta PQR$ (ASA)

(ii) (AAS Property)



$\angle BAC = \angle QPR$
$\angle ABC = \angle PQR$
$BC = QR$
$\therefore \Delta ABC \equiv \Delta PQR$ (AAS)

(d) The corresponding hypotenuse and a corresponding side of both right-angled triangles are equal. (RHS Property)



$\angle ABC = \angle PQR = 90^\circ$
$AC = PR$
$AB = PQ$
$\therefore \Delta ABC \equiv \Delta PQR$ (RHS)

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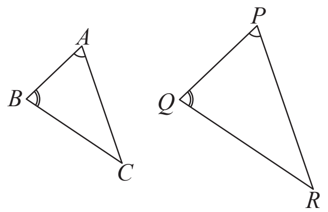
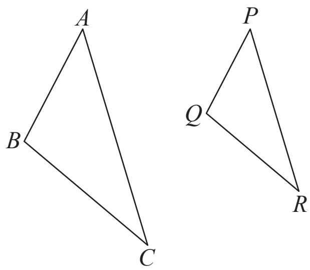
2. Test for Similarity

(a) The 2 corresponding angles of both triangles are equal. (AAA Property)



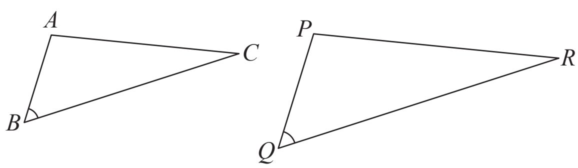
$\angle ABC = \angle PQR$
$\angle BAC = \angle QPR$
$\angle ACB = \angle PRQ$

(b) The 3 corresponding sides of the 2 triangles are proportional.



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

(c) The 2 corresponding sides of the 2 triangles are proportional and the included angle of one triangle is equal to the included angle of the other triangle.



$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\angle ABC = \angle PQR$

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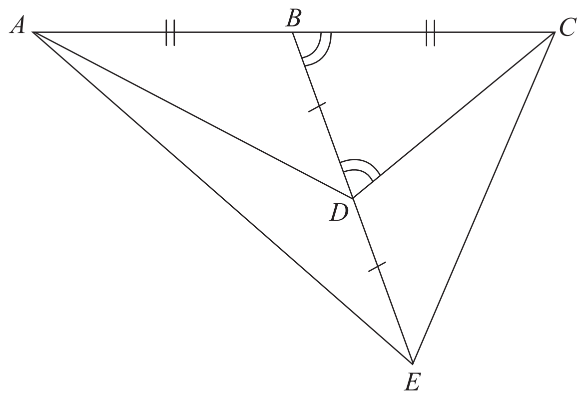
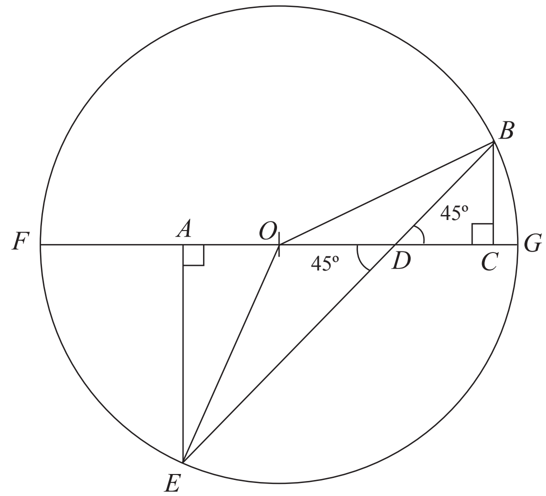
PRACTICE QUESTIONS

1. In the figure, *ABC* and *BDE* are straight lines. $\angle CBD = \angle CDB$. Prove that ΔABD is congruent to ΔCDE.



2. In the figure, *FG* is the diameter of a circle, centre *O*. *BDE* is a straight line. Prove that ΔAEO and ΔCOB are congruent.



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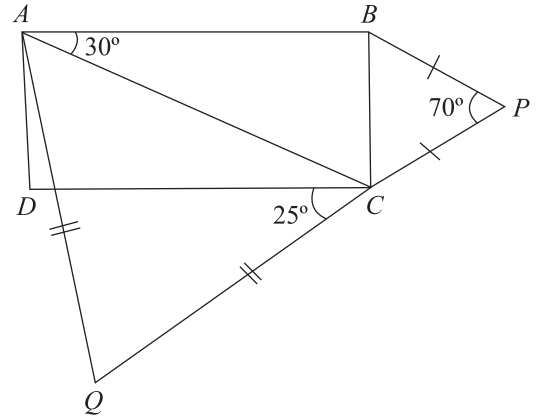
3. The diagram shows a rectangle $ABCD$ and two isosceles triangles PBC and ACQ. Given that $BP = CP$, $AQ = CQ$, $\angle BPC = 70^\circ$, $\angle BAC = 30^\circ$ and $\angle DCQ = 25^\circ$.

(a) Find $\angle PBC$.

(b) Show that triangles BCP and CAQ are similar.

(c) Prove that $\angle DAQ = 5^\circ$.



Solutions:

1. $\angle CBD = \angle CDB$ (given)

$\therefore$ $CB = CD$

$CB = BA$ (given)

$\therefore$ $CD = BA$

$BD = DE$ (given)

$\angle ABD = 180^\circ - \angle CBD$ (adj. ∠s on a st. line)

$= 180^\circ - \angle CDB$

$= \angle CDE$

$\therefore$ ΔABD is congruent to ΔCDE. (SAS)

2. $EO = OB$ (radii of the same circle)

$\angle OEB = \angle OBE$ (base ∠s of isos. Δ)

$\angle AEO = \angle AED - \angle OEB$

$= (180^\circ - 90^\circ - 45^\circ) - \angle OEB$

$= 45^\circ - \angle OEB$

$= 45^\circ - \angle OBE$

$= \angle COB$

In ΔAEO and ΔCOB,

$\angle AEO = \angle COB$ (proven)

$\angle OAE = \angle BCO = 90^\circ$ (given)

$EO = OB$ (proven)

$\therefore$ ΔAEO and ΔCOB are congruent. (AAS)

3. (a) $\angle PBC = \dfrac{180^\circ - 70^\circ}{2}$ (base ∠ of isos. Δ)

$= 55^\circ$

(b) $\angle ACD = \angle BAC$ (alt. ∠s, $AB//DC$)

$= 30^\circ$

$\angle ACQ = 30^\circ + 25^\circ$ (adj. ∠s)

Consider ΔBCP and ΔCAQ,

$\angle BCP = \angle CAQ = 55^\circ$

$\angle CBD = \angle ACQ = 55^\circ$

$\therefore$ ΔBCP and ΔCAQ are similar. (AA)

(c) $\angle DAQ = 90^\circ - 30^\circ - 55^\circ$ (adj. ∠s)

$= 5^\circ$ (proven)

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