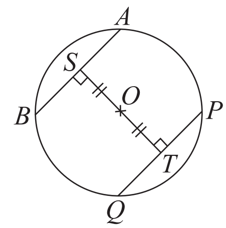
Geometrical Properties of Circles

Symmetry Properties of Circles

(i) Equal chords are equidistant from the centre

If $AB = PQ$,

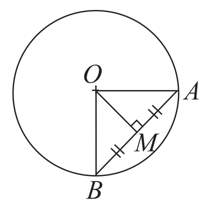
then $OS = OT$



(ii) The perpendicular bisector of a chord passes through the centre

If $OA = OB$,
$AM = MB$,

then OM is perpendicular to AB



(iii) Tangents from an external point are equal in length

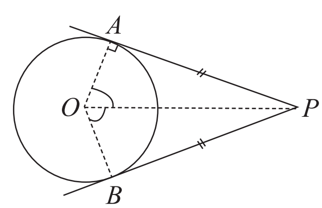
If AP and BP are tangents,

then $AP = BP$

(iv) The line joining an external point to the centre of the circle bisects the angle between the tangents

If $AP = BP$,

then $\angle AOP = \angle BOP$



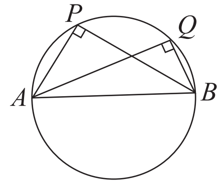
Angle Properties of Circles

(i) Angle in a semicircle is a right angle

If AB is the diameter of a circle,

then $\angle APB = 90^\circ$

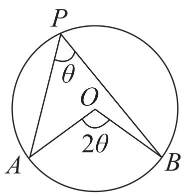
$\angle AQB = 90^\circ$

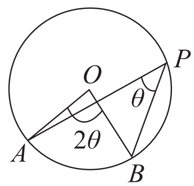


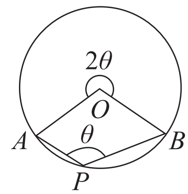
(ii) Angle at the centre is twice the angle at the circumference ($\angle$ at centre = 2 × $\angle$ at ⊙^{ce})

If O is the centre of the circle,

then $\angle AOB = 2 \times \angle APB$



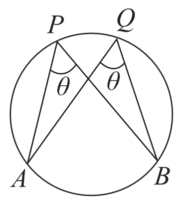




(iii) Angles in the same segment are equal

If P and Q are points on the same segment,

then $\angle APB = \angle AQB$ ($\angle$s in same segment)

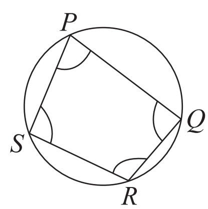


(iv) Angles in opposite segments are supplementary

If P, Q, R and S are points on the circumference of a circle,

then $\angle PQR + \angle PSR = 180^\circ$ ($\angle$s in opp. segments are supp.)

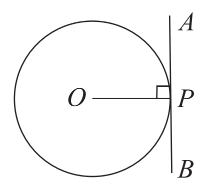
$\angle QRS + \angle QPS = 180^\circ$ ($\angle$s in opp. segments are supp.)



(v) Angle between tangent and radius of a circle is a right angle

If O is the centre of a circle and AB is the tangent at point P,

then OP is perpendicular to AB. (tan ⊥ rad)



Adapted:
Be Great at Mathematics Secondary 3 & 4

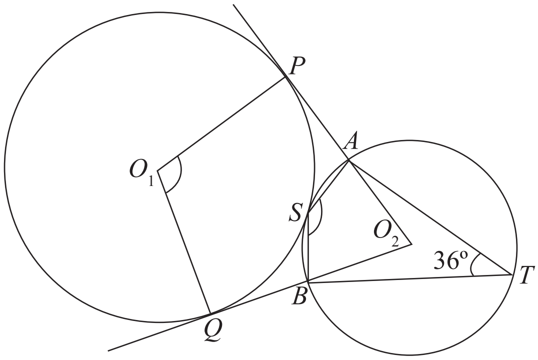
PRACTICE QUESTIONS

1. In the diagram, O_1 and O_2 are the centres of the circles. PO_2 and QO_2 are tangents to the bigger circle at points P and Q respectively and $\angle ATB = 36°$.

Find

(a) $\angle ASB$,

(b) $\angle PO_1Q$.



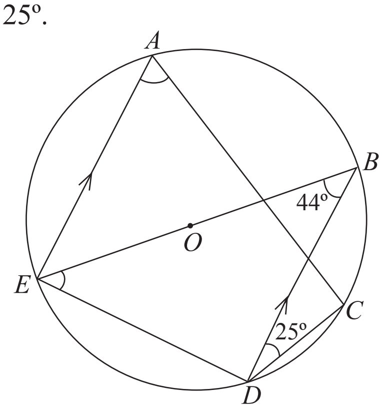
2. In the diagram, A, B, C, D and E are the points on the circumference of the circle with centre O. AE and BD are parallel, $\angle EBD = 44°$ and $\angle BDC = 25°$.

(a) Find

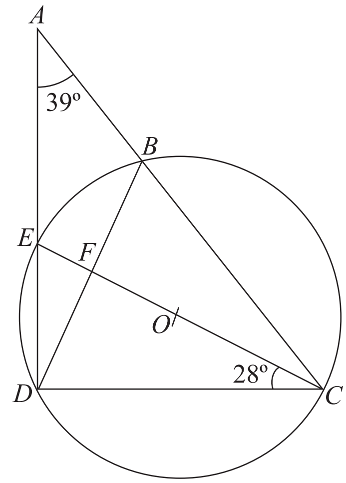
(i) $\angle BED$,

(ii) $\angle CAE$.

(b) Show that AD is the diameter of the circle.



3. In the figure, O is the centre of the circle. ABC, AED, BFD and CFE are straight lines. Calculate $\angle BDC$.



Solutions:

1. (a) $\angle ASB = 180^\circ - 36^\circ$ ($\angle$s in opp. segments are supp.)
$= 144^\circ$
$\angle AO_2B = 2(36^\circ)$ ($\angle$ at centre = 2 $\angle$s at $\odot^{ce}$)
$= 72^\circ$
$\angle PO_1Q = 360^\circ - 90^\circ - 90^\circ - 72^\circ$ (tan $\perp$ rad, $\angle$ sum of quad.)
$= 108^\circ$

2. (a) (i) $\angle BED = 180^\circ - 44^\circ - 90^\circ$
$= 46^\circ$ ($\angle$ sum of rt. $\angle\Delta$)
(ii) $\angle CAE = 180^\circ - 90^\circ - 25^\circ$ ($\angle$s in opp. segments are supp.)
$= 65^\circ$
(b) $\angle EOD = 2(44^\circ)$
$= 88^\circ$
$\angle AOE = 180^\circ - 2(44^\circ)$ ($\angle$ sum of isos. Δ)
$= 92^\circ$
$\angle AOE + \angle EOD = 92^\circ + 88^\circ$
$= 180^\circ$
AD is a diameter as AOD is a straight line and $\angle AED = 90^\circ$. (shown)

3. In ΔACD,
$\angle ADC = 90^\circ$ ($\angle$ in semicircle)
$\angle CAD = 39^\circ$ (given)
$\angle ACD = 180^\circ - 90^\circ - 39^\circ$ ($\angle$ sum of Δ)
$= 51^\circ$
$\angle BCE = \angle ACD - \angle DCE$
$= 51^\circ - 28^\circ$
$= 23^\circ$
$\angle BDE = \angle BCE$ ($\angle$s in the same segment)
$= 23^\circ$
$\angle BDC = \angle ADC - \angle BDE$
$= 90^\circ - 23^\circ$
$= 67^\circ$