

Lowest Common Multiple

Classic Example

Alan goes to the library once every 3 days.
Ben does the same once every 5 days.
Chloe goes to the library once every 7 days.
The last time they met at the library was 5th of January.
Find the date when the three of them will meet again.
[Hint: That February has 28 days.]

Solution

The numbers 3, 5 and 7 do not have common factors or divisors, hence we multiply the three numbers.

Lowest Common Multiple $\rightarrow 3 \times 5 \times 7 = 105$ days

$$105 - (31 - 5) - 28 - 31 = 20$$

5 days gone for Jan ↑ Feb ↑ Mar

Ans: 20th of April

Comment

Questions of this nature bring out the topic of Lowest Common Multiple, or LCM.

Notes

- (i) A prime number has only two factors, 1 and itself. Some of the prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, ... etc.
- (ii) Students can practise on the following using division by prime to gain familiarity with LCM.

- (a) Find the LCM of 4, 5, 6.

Step (i): Divide the three numbers by a prime number.

$$\begin{array}{r|rrr} 2 & 4 & 5 & 6 \\ \hline & 2 & 5 & 3 \end{array}$$

Step (ii): There are no more common factors for 2, 5 and 3.

Step (iii): Multiply the divisor and the quotients.

$$\begin{array}{r|rrr} \textcircled{2} & 4 & 5 & 6 \\ \hline & \textcircled{2} & \textcircled{5} & \textcircled{3} \end{array}$$

$$2 \times 2 \times 5 \times 3 = \mathbf{60}$$

- (b) Find the LCM of 12, 18, 24.

Step (i): Keep dividing the three numbers by a prime factor.

$$\begin{array}{r|rrr} \textcircled{2} & 12 & 18 & 24 \\ \hline \textcircled{3} & 6 & 9 & 12 \\ \hline \textcircled{2} & 2 & 3 & 4 \\ \hline & \textcircled{1} & \textcircled{3} & \textcircled{2} \end{array}$$

Step (ii): 1, 3 and 2 do not have a common divisor.

Step (iii): Multiply the divisors and the quotients.

$$2 \times 3 \times 2 \times 1 \times 3 \times 2 = \mathbf{72}$$

Practice

1 Find the lowest common multiple (LCM) for each of the following.

(a) 5, 6, 7

Ans: (a) _____

(b) 3, 8, 12

Ans: (b) _____

(c) 4, 7, 12

Ans: (c) _____

(d) 5, 18, 25

Ans: (d) _____

(e) 7, 15, 18

Ans: (e) _____

(f) 5, 21, 35

Ans: (f) _____

(g) When a pack of bite-sized chocolates is shared equally among 3, 4, 5 or 6 children, there is always a remainder of 2 chocolates.

Find the smallest possible number of chocolates in the pack.

Ans: (g) _____

2 Bus 42 leaves the terminal every 10 minutes.

Bus 52 and Bus 62 leave the terminal every 15 and 20 minutes respectively.

If all three buses leave the terminal simultaneously at 12 10 h, at what time will they leave the terminal at the same time again?

Ans: _____ h

Solutions:

Lowest Common Multiple

1 (a) $5 \times 6 \times 7 = 210$

(b)
$$\begin{array}{r|rrr} 3 & 3 & 8 & 12 \\ \hline 2 & 1 & 8 & 4 \\ \hline 2 & 1 & 4 & 2 \\ \hline & 1 & 2 & 1 \end{array}$$

$3 \times 2 \times 2 \times 2 = 24$

(c)
$$\begin{array}{r|rrr} 2 & 4 & 7 & 12 \\ \hline 2 & 2 & 7 & 6 \\ \hline & 1 & 7 & 3 \end{array}$$

$2 \times 2 \times 7 \times 3 = 84$

(d)
$$\begin{array}{r|rrr} 5 & 5 & 18 & 25 \\ \hline & 1 & 18 & 5 \end{array}$$

$5 \times 18 \times 5 = 450$

(e)
$$\begin{array}{r|rrr} 3 & 7 & 15 & 18 \\ \hline & 7 & 5 & 6 \end{array}$$

$3 \times 7 \times 5 \times 6 = 630$

(f)
$$\begin{array}{r|rrr} 5 & 5 & 21 & 35 \\ \hline 7 & 1 & 21 & 7 \\ \hline & 1 & 3 & 1 \end{array}$$

$5 \times 7 \times 3 = 105$

(g)
$$\begin{array}{r|rrrr} 3 & 3 & 4 & 5 & 6 \\ \hline 2 & 1 & 4 & 5 & 2 \\ \hline & 1 & 2 & 5 & 1 \end{array}$$

$3 \times 2 \times 2 \times 5 = 60$

$60 + 2 = 62$

Ans: 62 chocolates

2
$$\begin{array}{r|rrr} 5 & 10 & 15 & 20 \\ \hline 2 & 2 & 3 & 4 \\ \hline & 1 & 3 & 2 \end{array}$$

$5 \times 2 \times 3 \times 2 = 60$ min

1 h after 1210 h

Ans: 1310 h