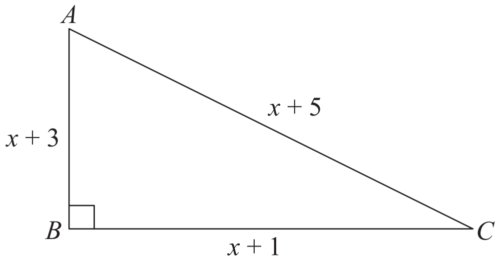
Pythagoras' Theorem

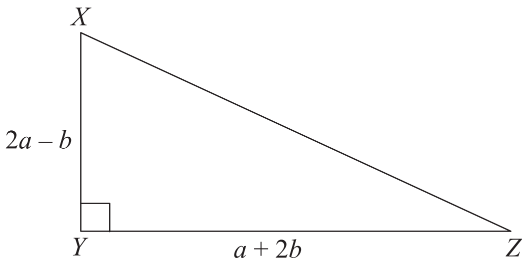
1. In the figure, $\triangle ABC$ is a right-angled triangle such that $AB = (x + 3)$ cm, $BC = (x + 1)$ cm, $AC = (x + 5)$ cm and $\angle ABC = 90°$. Find the value of x.



2. $\triangle XYZ$ is a right-angled triangle such that $XY = (2a – b)$ cm, $YZ = (a + 2b)$ cm and $\angle XYZ = 90°$.

(a) Show that $XZ^2 = 5(a^2 + b^2)$.

(b) Hence, given that $(a + b)^2 = 141$ and $ab = 8$, find the length XZ.



Solutions to

Pythagoras' Theorem

1. Since $\triangle ABC$ is right-angled, by the Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$
$$(x + 5)^2 = (x + 3)^2 + (x + 1)^2$$
$$x^2 + 10x + 25 = x^2 + 6x + 9 + x^2 + 2x + 1$$
$$x^2 + 10x + 25 = 2x^2 + 8x + 10$$
$$x^2 - 2x - 15 = 0$$
$$(x - 5)(x + 3) = 0$$
$$x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = 5 \quad \text{or} \quad x = -3 \ \text{(rej.)}$$
$$\therefore x = 5$$

Note: $x = -3$ is rejected because $BC = -3 + 1 = -2$ cm, which is impossible.

2. (a) Since $\triangle XYZ$ is a right-angled triangle, by the Pythagoras' Theorem,

$$XZ^2 = (2a - b)^2 + (a + 2b)^2$$
$$= (2a^2) - 2(2a)(b) + b^2 + a^2 + 2(a)(2b) + (2b)^2$$
$$= 4a^2 - 4ab + b^2 + a^2 + 4ab + 4b^2$$
$$= 5a^2 - 4ab + 5b^2 + 4ab$$
$$= 5a^2 + 5b^2$$
$$= 5(a^2 + b^2) \qquad \text{(shown)}$$

(b)

$$(a + b)^2 = 141$$
$$a^2 + b^2 + 2ab = 141$$
$$a^2 + b^2 + 2(8) = 141$$
$$a^2 + b^2 = 125$$
$$\therefore XZ^2 = 5(a^2 + b^2)$$
$$= 5(125)$$
$$= 625$$
$$XZ = \sqrt{625} \ \text{or} \ -\sqrt{625} \ \text{(rej.)}$$
$$= 25 \text{ cm}$$