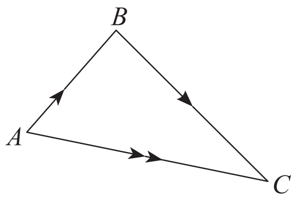
Vectors

- **Magnitude of a Column Vector**

  For $\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$,

  magnitude of $\boldsymbol{a} = |\boldsymbol{a}| = \sqrt{x^2 + y^2}$

- **Triangle Law of Addition**

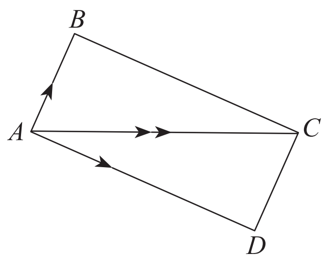


$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

- **Subtraction of Vectors**

  $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$

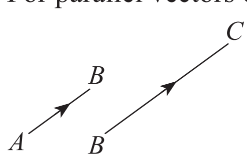
- **Parallelogram Law of Addition**



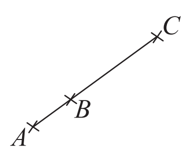
$ABCD$ is a parallelogram.
$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

- **Multiplying a Vector by a Scalar**

  For parallel vectors or points that are collinear, $\overrightarrow{AB} = k\overrightarrow{BC}$



or



- **Ratio of Areas of Triangles**

  Method 1: If both triangles are similar,

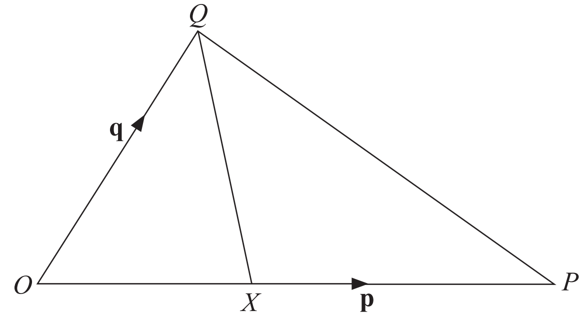
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

  Method 2: If both triangles share a common height,

$$\frac{A_1}{A_2} = \frac{\frac{1}{2} \times b_1 \times h}{\frac{1}{2} \times b_2 \times h} = \frac{b_1}{b_2}$$

PRACTICE QUESTIONS

1. In the figure below, $\overrightarrow{OP}$ = p and $\overrightarrow{OQ} = \mathbf{q}$. If $\overrightarrow{OX} = \frac{3}{5}\overrightarrow{OP}$, find $\overrightarrow{QX}$ in terms of **p** and **q**.



2. In the figure below, *OABC* is a parallelogram.

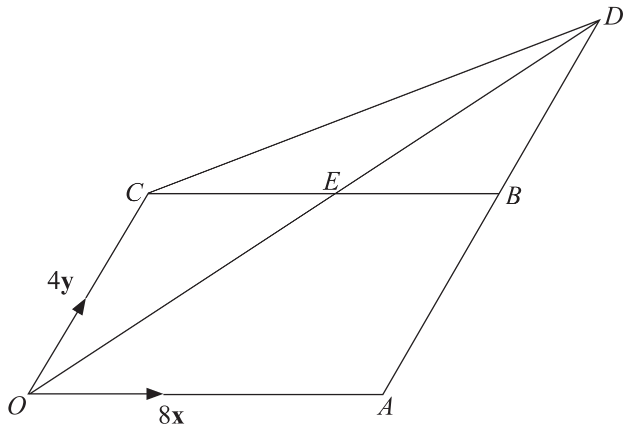
The vectors $\overrightarrow{OA} = 8\mathbf{x}$ and $\overrightarrow{OC} = 4\mathbf{y}$. The lines $AB = 2BD$ and $CE = EB$.

(a) Express the following vectors in terms of **x** and **y**.

(i) $\overrightarrow{AC}$

(ii) $\overrightarrow{OD}$

(b) Find the value of the ratio of the area of triangle *OCE* to the area of triangle *OAD*.

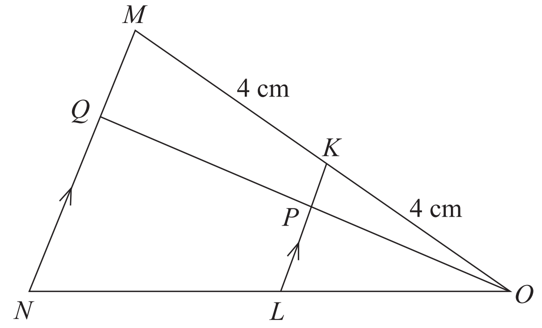


3. In the diagram, $OK = MK = 4$ cm, MN is parallel to KL, OQ and KL intersect at P and $\frac{KP}{PL} = \frac{1}{3}$.

(a) Name a triangle which is similar to ΔOMN.

(b) Write down the numerical value of

(i) $\frac{\text{area of } \Delta OKP}{\text{area of } \Delta OKL}$, (ii) $\frac{\text{area of } \Delta OKL}{\text{area of } \Delta OMN}$.



Solutions:

1. $\overrightarrow{OX} = \frac{3}{5}\overrightarrow{OP} = \frac{3}{5}\mathbf{p}$

$\overrightarrow{QX} = -\overrightarrow{OX} + \overrightarrow{OQ} = \mathbf{q} - \frac{3}{5}\mathbf{p}$

2. (a) (i) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\mathbf{y} - 8\mathbf{x}$

(ii) Since $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$,

$\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AB} = \frac{3}{2}(4\mathbf{y}) = 6\mathbf{y}$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = 8\mathbf{x} + 6\mathbf{y}$

(b) Triangle OCE is similar to triangle OAD.

$\therefore$ Ratio of area of $\frac{OCE}{OAD} = \frac{4\mathbf{y} \times 4\mathbf{x}}{6\mathbf{y} \times 8\mathbf{x}} = \frac{1}{3}$

3. (a) ΔOKL

(b) (i) $\frac{\text{Area of } \Delta OKP}{\text{Area of } \Delta OKL}$

$= \frac{\frac{1}{2} \times KP \times \text{height}}{\frac{1}{2} \times KL \times \text{height}}$

$= \frac{KP}{KL}$

$= \frac{1}{1+3}$

$= \frac{1}{4}$

(ii) $\frac{\text{Area of } \Delta OKL}{\text{Area of } \Delta OMN} = \left(\frac{OK}{OM}\right)^2$

$= \left(\frac{4}{8}\right)^2$

$= \frac{1}{4}$