

$$\begin{aligned}
 1. \quad \frac{1}{4 + \frac{1}{5}} &= \frac{1}{\frac{20+1}{5}} = \frac{1}{\frac{21}{5}} = \frac{5}{21} \\
 \frac{1}{3 + \frac{5}{21}} &= \frac{1}{\frac{63+5}{21}} = \frac{1}{\frac{68}{21}} = \frac{21}{68} \\
 \frac{1}{2 + \frac{21}{68}} &= \frac{1}{\frac{136+21}{68}} = \frac{1}{\frac{157}{68}} = \frac{68}{157}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{1 + \frac{1}{8-1}}{1 + \frac{1}{3+1}} &= \frac{1 + \frac{1}{7}}{1 + \frac{1}{4}} = \frac{1 + \frac{4}{7}}{1 + \frac{3}{4}} = \frac{\frac{7+4}{7}}{\frac{4+3}{4}} \\
 &= \frac{\frac{11}{7}}{\frac{7}{4}} = \frac{11}{7} \times \frac{4}{7} = \frac{44}{49}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{1}{2 + \frac{1}{7}} &= \frac{1}{\frac{14+1}{7}} = \frac{1}{\frac{15}{7}} = \frac{7}{15} \\
 \frac{1}{3 - \frac{7}{15}} &= \frac{1}{\frac{45-7}{15}} = \frac{1}{\frac{38}{15}} = \frac{15}{38} \\
 \frac{1}{4 + \frac{15}{38}} &= \frac{1}{\frac{152+15}{38}} = \frac{38}{167}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad 1 + \frac{1}{3} - (\frac{1}{3} + \frac{1}{4}) + (\frac{1}{4} + \frac{1}{5}) - (\frac{1}{5} + \frac{1}{6}) \\
 + (\frac{1}{6} + \frac{1}{7}) - (\frac{1}{7} + \frac{1}{8}) + (\frac{1}{8} + \frac{1}{9}) \\
 = 1 + \frac{1}{3} - \frac{1}{3} - \frac{1}{4} + \frac{1}{4} + \frac{1}{5} - \dots + \frac{1}{8} + \frac{1}{9} \\
 = 1 + \frac{1}{9} \\
 = 1\frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad (1 - \frac{1}{2}) + (1 - \frac{1}{6}) + (1 - \frac{1}{12}) + (1 - \frac{1}{20}) + \dots + (1 - \frac{1}{90}) \\
 = 9 - (\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{90}) \\
 = 9 - (1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{9} - \frac{1}{10}) \\
 = 9 - (1 - \frac{1}{10}) \\
 = 9 - 1 + \frac{1}{10} \\
 = 8\frac{1}{10}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \frac{1}{4} \times (1 - \frac{1}{5} + \frac{1}{5} - \frac{1}{9} + \frac{1}{9} - \frac{1}{13} + \dots + \frac{1}{97} - \frac{1}{101}) \\
 = \frac{1}{4} \times (1 - \frac{1}{101}) \\
 = \frac{1}{4} \times \frac{100}{101} \\
 = \frac{25}{101}
 \end{aligned}$$

7. *Analysis: Group fractions of the same denominator.*

$$\begin{aligned}
 \frac{1}{2} + (\frac{1}{3} + \frac{2}{3}) + (\frac{1}{4} + \frac{2}{4} + \frac{3}{4}) + \dots + (\frac{1}{10} + \dots + \frac{9}{10}) \\
 = \frac{1}{2} + 1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + \dots + 4\frac{1}{2} \\
 4 \text{ pairs of } 5, 2\frac{1}{2} \text{ is not paired.} \\
 = 4 \times 5 + 2\frac{1}{2} \\
 = 20 + 2\frac{1}{2} \\
 = 22\frac{1}{2}
 \end{aligned}$$

$$8. \quad \frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \dots + \frac{1}{13 \times 15} \quad \text{---(1)}$$

Recall $\frac{1}{n \times (n+d)} = \frac{1}{d} \times (\frac{1}{n} - \frac{1}{n+d})$.

(1) becomes

$$\begin{aligned}
 \frac{1}{2} \times (1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{13} - \frac{1}{15}) \\
 = \frac{1}{2} \times (1 - \frac{1}{15}) \\
 = \frac{1}{2} \times \frac{14}{15} \\
 = \frac{14}{30} \\
 = \frac{7}{15}
 \end{aligned}$$

$$9. \quad \text{Let } A \text{ be } \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}.$$

$$\begin{aligned}
 A^2 + \frac{1}{2}A - (1+A) \times (A - \frac{1}{2}) \\
 = A^2 + \frac{1}{2}A - (A - \frac{1}{2} + A^2 - \frac{1}{2}A) \\
 = \cancel{A^2} + \frac{1}{2}\cancel{A} - \cancel{A} + \frac{1}{2} - \cancel{A^2} + \frac{1}{2}\cancel{A} \\
 = \frac{1}{2}
 \end{aligned}$$