

Differentiation

Key Concepts

Definition

- 1 Differentiation is the process of obtaining the derivative, $\frac{dy}{dx}$, of a given function.
- 2 $\frac{dy}{dx}$ is also known as the derived function or differential coefficient of y with respect to x .

Derivatives

- 3 If $y = f(x)$, the first derivative of y with respect to x can be written in 3 different ways as shown below.
 - (a) $\frac{dy}{dx}$
 - (b) $f'(x)$
 - (c) $\frac{d}{dx} [f(x)]$
- 4 The second derivative of $y = f(x)$ can be written in 3 different ways as shown below.
 - (a) $\frac{d^2y}{dx^2}$
 - (b) $f''(x)$
 - (c) $\frac{d}{dx} \left(\frac{dy}{dx} \right)$
- 5 The way of writing the derivatives of y with respect to x depends on what form is given, as shown in the table below.

Given	Example	1 st derivative	2 nd derivative
y	$y = x^3 + 5x$	$\frac{dy}{dx} = 3x^2 + 5$	$\frac{d^2y}{dx^2} = 6x$
$f(x)$	$f(x) = x^3 + 5x$	$f'(x) = 3x^2 + 5$	$f''(x) = 6x$
Expression	$x^3 + 5x$	$\frac{d}{dx} (x^3 + 5x) = 3x^2 + 5$	$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (3x^2 + 5) = 6x$

Formulae

In all the formulae below, k and n are constants.

- 6 Differentiation of functions with powers of x

$$\frac{d}{dx}(kx^n) = knx^{n-1}$$

- 7 Differentiation of trigonometric functions (x in radians)

(a) $\frac{d}{dx}(k \sin x) = k \cos x$

(b) $\frac{d}{dx}(k \cos x) = -k \sin x$

(c) $\frac{d}{dx}(k \tan x) = k \sec^2 x$

- 8 Differentiation of exponential functions

$$\frac{d}{dx}(ke^x) = ke^x$$

Differentiation of a Composite Function (Function of a Function)

- 9 A composite function can be differentiated using the chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

- 10 The chain rule is applied in the differentiation of the following composite functions.

- (a) powers of x
- (b) trigonometric
- (c) exponential
- (d) logarithmic

- 11 Differentiation of composite functions with powers of x

(a) $\frac{d}{dx}[(ax + b)^n] = an(ax + b)^{n-1}$

(b) In general, $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} f'(x)$

- 12 Differentiation of composite trigonometric functions (x in radians)

(a) (i) $\frac{d}{dx}[k \sin(ax + b)] = ak \cos(ax + b)$

(ii) $\frac{d}{dx}[k \cos(ax + b)] = -ak \sin(ax + b)$

(iii) $\frac{d}{dx}[k \tan(ax + b)] = ak \sec^2(ax + b)$

- (b) In general,

(i) $\frac{d}{dx}[k \sin f(x)] = k [\cos f(x)] \cdot f'(x)$

(ii) $\frac{d}{dx}[k \cos f(x)] = -k [\sin f(x)] \cdot f'(x)$

(iii) $\frac{d}{dx}[k \tan f(x)] = k [\sec^2 f(x)] \cdot f'(x)$

Adapted:

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- (c) (i) $\frac{d}{dx} [k \sin^n(ax + b)] = akn \sin^{n-1}(ax + b) \cos(ax + b)$
 (ii) $\frac{d}{dx} [k \cos^n(ax + b)] = -akn \cos^{n-1}(ax + b) \sin(ax + b)$
 (iii) $\frac{d}{dx} [k \tan^n(ax + b)] = akn \tan^{n-1}(ax + b) \sec^2(ax + b)$

13 Differentiation of composite exponential functions

- (a) $\frac{d}{dx} [ke^{ax+b}] = ake^{ax+b}$
 (b) In general, $\frac{d}{dx} [ke^{f(x)}] = kf'(x)e^{f(x)}$

14 Differentiation of logarithmic functions

- (a) $\frac{d}{dx} [k \ln x] = \frac{k}{x}$
 (b) $\frac{d}{dx} [k \ln(ax + b)] = \frac{ak}{ax + b}$
 (c) In general, $\frac{d}{dx} [\ln f(x)] = \frac{f'(x)}{f(x)}$

Differentiation of a Product (Product Rule)

15 If $y = uv$ where u and v are functions of x , then

$$\frac{dy}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$$

Differentiation of a Quotient (Quotient Rule)

16 If $y = \frac{u}{v}$ where u and v are functions of x , then

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$