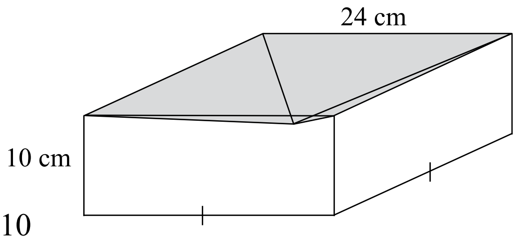
Volume and Surface Area of Pyramids, Cones and Spheres

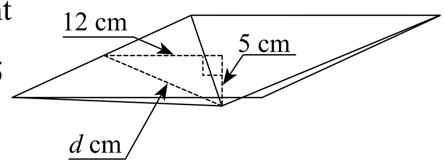
Example 1

A right pyramid with a square base is removed from a rectangular wooden block as shown in the figure.
The height of the pyramid is half of the wooden block.
Find
(a) the volume,
(b) the total surface area
of the remaining wooden block.



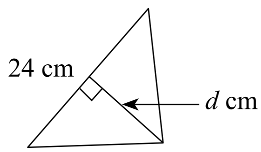
Solution: (a) Height of the pyramid $= \frac{1}{2} \times 10$
$= 5$ cm

Volume of the remaining wooden block
= volume of the block − volume of the pyramid
$= l \times b \times h - \frac{1}{3} \times \text{base area} \times \text{height}$
$= 24 \times 24 \times 10 - \frac{1}{3} \times (24 \times 24) \times 5$
$= 4800 \text{ cm}^3$



(b) Using Pythagoras' theorem,
$d^2 = 12^2 + 5^2$
$= 169$
$d = \sqrt{169} \quad (d > 0)$
$= 13$ cm

Total area of the slant faces of the pyramid
= 4 × area of the triangle
$= 4 \times \frac{1}{2} \times 24 \times 13$
$= 624 \text{ cm}^2$



Total area of the vertical faces $= 4(10 \times 24)$
$= 960 \text{ cm}^2$

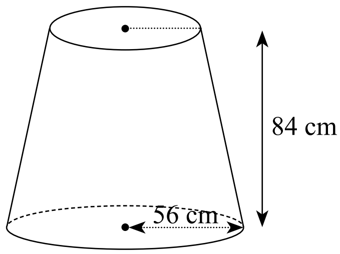
Area of the base $= 24 \times 24$
$= 576 \text{ cm}^2$

Total surface area of the remaining wooden block $= 624 + 960 + 576$
$= 2160 \text{ cm}^2$

Example

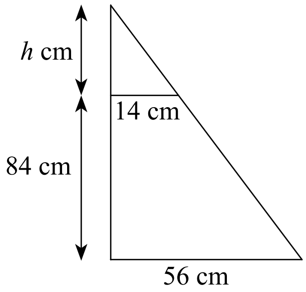
2 A symmetrical solid has a circular base of radius 56 cm and a circular top. The radius of the circular top is $\frac{1}{4}$ of the circular base. Given that the height of the solid is 84 cm, find its volume.

[Take $\pi = \frac{22}{7}$]



Solution: Radius of the circular top $= \frac{1}{4} \times 56$

$= 14$ cm



Using similar triangles, $\frac{h}{14} = \frac{h + 84}{56}$

$4h = h + 84$

$3h = 84$

$h = 28$ cm

Volume of the solid = volume of the bigger cone − volume of the smaller cone

$= \frac{1}{3} \times \frac{22}{7} \times 56^2 \times (84 + 28) - \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 28$

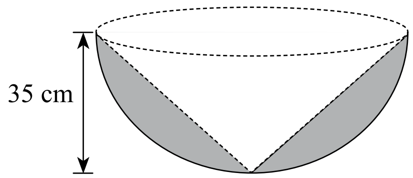
$= 362\ 208$

$\approx 362\ 000\ \text{cm}^3$ (3 s.f.)

Example 3

A right circular cone is removed from a metal hemisphere as shown in the figure. If 1 cm^3 of the metal weighs 5.78 g, calculate the mass of the remaining solid, correcting to the nearest kilogram.

[Take $\pi = \frac{22}{7}$]



Solution: Volume of the hemisphere $= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} \times 35^3$

$= 89\ 833\frac{1}{3}$ cm^3

Volume of the cone (removed) $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 35^2 \times 35$

$= 44\ 916\frac{2}{3}$ cm^3

Volume of the remaining solid = volume of the hemisphere – volume of the cone

$= 89\ 833\frac{1}{3} - 44\ 916\frac{2}{3}$

$= 44\ 916\frac{2}{3}$ cm^3

Mass of the remaining solid = density × volume of the remaining solid

$= 5.78$ g/cm^3 $\times 44\ 916\frac{2}{3}$ cm^3

$\approx 259\ 618\frac{1}{3}$ g

≈ 260 kg (nearest kg)

Adapted:
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