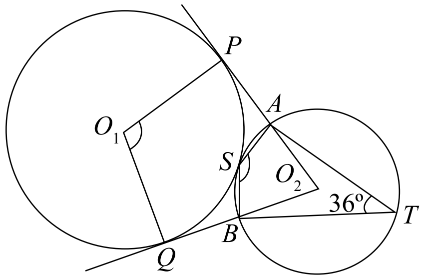
Geometrical Properties of Circles

Example 1

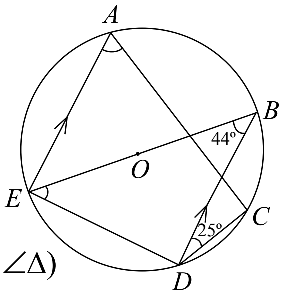
In the diagram, O_1 and O_2 are the centres of the circles. PO_2 and QO_2 are tangents to the bigger circle at point P and Q respectively and $\angle ATB = 36°$.
Find
(a) $\angle ASB$,
(b) $\angle PO_1Q$.



Solution: (a) $\angle ASB = 180° - 36°$ ($\angle$s in opp. segments are supp.)
$= 144°$
$\angle AO_2B = 2(36°)$ ($\angle$ at centre = 2 $\angle$s at $\odot^{ce}$)
$= 72°$
(b) $\angle PO_1Q = 360° - 90° - 90° - 72°$ (tan $\perp$ rad, sum $\angle$s of quad)
$= 108°$

Example 2

In the diagram, A, B, C, D and E are the points on the circumference of the circle, centre O. AE and BD are parallel, $\angle EBD = 44°$ and $\angle BDC = 25°$.
(a) Find (i) $\angle BED$,
(ii) $\angle CAE$.
(b) Show that AD is the diameter of the circle.



Solution: (a) (i) $\angle BED = 180° - 44° - 90°$ (sum $\angle$s of rt. $\angle\Delta$)
$= 46°$
(ii) $\angle CAE = 180° - 90° - 25°$ ($\angle$s in opp. segments are supp.)
$= 65°$
(b) $\angle EOD = 2(44°)$
$= 88°$
$\angle AOE = 180° - 2(44°)$
$= 92°$
$\angle AOE + \angle EOD = 92° + 88°$
$= 180°$
AD is a diameter as AOD is a straight line and $\angle AED = 90°$. (shown)

Adapted:
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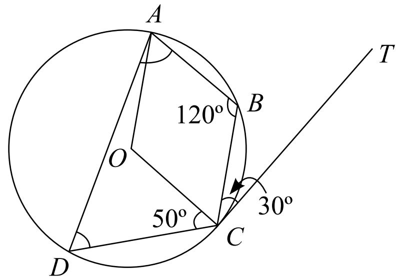
Example 3

In the diagram, A, B, C and D are the points on the circumference of the circle, centre O. CT is a tangent to the circle at point C, $\angle ABC = 120°$, $\angle BCT = 30°$ and $\angle OCD = 50°$. Find

(a) $\angle BAD$.

(b) $\angle ADC$.



Solution: (a) $\angle OCB = 90° - 30°$ (tan ⊥ rad)

$= 60°$

$\angle BAD = 180° - 50° - 60°$ (∠s in opp. segments are supp.)

$= 70°$

(b) $\angle AOC = 180° - 120°$ (∠s in opp. segments are supp.)

$= 60°$

$\therefore \angle ADC = 30°$ (∠ at centre = 2 ∠s at ⊙ce)

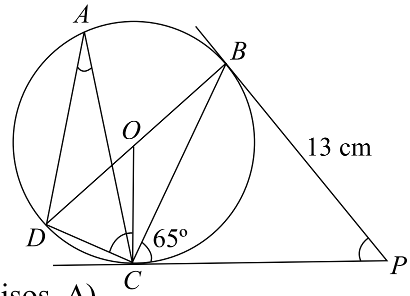
Example 4

In the diagram, A, B, C and D are the points on the circumference of the circle, centre O. BP and CP are tangents to the circle at points B and C respectively, $\angle BCP = 65°$ and $BP = 13$ cm.

Calculate

(a) $\angle BPC$,

(b) length of OB,



Solution: (a) Since $BP = CP$,

$\angle BPC = 180° - 2(65°)$ (sum ∠s of isos. Δ)

$= 30°$

(b) Consider ΔOBP, $\tan 15° = \dfrac{OB}{13}$

$OB = 3.48$ cm (3 sig. fig.)

(c) $\angle OCB = 90° - 65°$ (tan ⊥ rad)

$= 25°$

$\angle DOC = 2(25°)$ (ext ∠s)

$= 50°$

$\angle CAD = \dfrac{50°}{2}$ (∠ at centre = 2 ∠s at ⊙ce)

$= 25°$

(d) $\angle OCD = \dfrac{180° - 50°}{2}$

$= 65°$