Integration and Plane Area

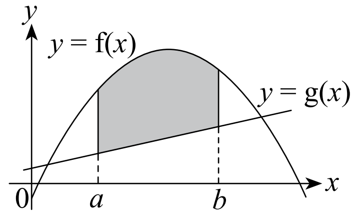
Key Concepts

Area Between Two Graphs and Vertical Boundaries x = a and x = b

1 The shaded area bounded by the two graphs and the vertical boundaries $x = a$ and $x = b$ is given by the formula

$A = \int_a^b [f(x) - g(x)]\, dx$

(i.e. integral of the function which forms the upper boundary subtract integral of the function which forms the lower boundary of the area).



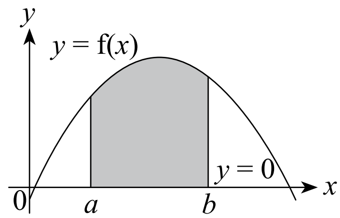
Area Between a Graph, the x-axis and Vertical Boundaries *x* = *a* and *x* = *b*

2 In the case where the area (above the x-axis) is bounded by a graph, the x-axis and the vertical boundaries $x = a$ and $x = b$, the formula simplifies to

$A = \int_a^b [f(x) - g(x)]\, dx$

$A = \int_a^b [f(x) - 0]\, dx$ since x-axis is $g(x) = 0$

$A = \int_a^b f(x)\, dx$

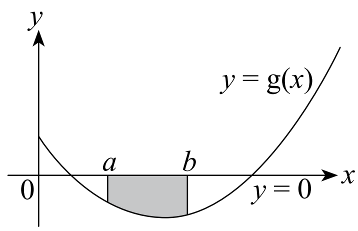


3 In the case where the area (below the x-axis) is bounded by a graph, the x-axis and the vertical boundaries $x = a$ and $x = b$, the formula simplifies to

$A = \int_a^b [0 - g(x)]\, dx$ since x-axis is $f(x) = 0$

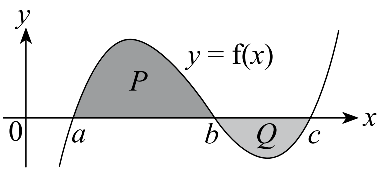
$A = \int_a^b - g(x)\, dx$

This can also be written as $A = \left|\int_a^b g(x)\, dx\right|$.



Area Between Two Graphs with Two Regions (one region above the x-axis and the other region below the x-axis)

4 In the case where the area bounded by a graph and the x-axis has a region P above the x-axis and a region Q below the x-axis, the area of the two regions must be found by integrating separately (i.e. a combination of the formulae in Key Concepts 2 and 3) as follows:



$A = \int_a^b f(x)\,dx + \int_b^c -f(x)\,dx$

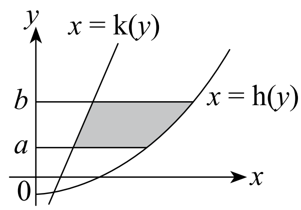
or $A = \int_a^b f(x)\,dx + \left|\int_b^c f(x)\,dx\right|$

and not $A = \int_a^c f(x)\,dx$, which will result in area of region Q subtracted from the area of region P.

Area Between Two Graphs and Horizontal Boundaries *y* = *a* and *y* = *b*

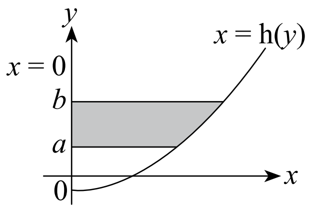
5 The shaded area bounded by the graphs and the horizontal boundaries $y = a$ and $y = b$ is given by the formula $A = \int_a^b [h(y) - k(y)]\,dy$

(i.e. integral of the function which forms the right boundary subtract integral of the function which forms the left boundary of the area).



Area Between a Graph, the *y*-axis and Horizontal Boundaries *y* = *a* and *y* = *b*

6 In the case where the area (right of the y-axis) is bounded by a graph, the y-axis and the horizontal boundaries $y = a$ and $y = b$, the formula simplifies to

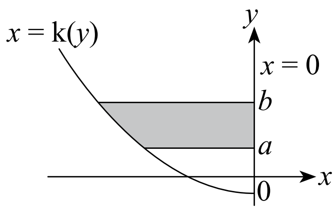


$A = \int_a^b [h(y) - k(y)]\,dy$

$A = \int_a^b [h(y) - 0]\,dy$ since y-axis is $k(y) = 0$

$A = \int_a^b h(y)\,dy$

7 In the case where the area (left of the y-axis) is bounded by a graph, the y-axis and the horizontal boundaries $y = a$ and $y = b$, the formula simplifies to



$A = \int_a^b [h(y) - k(y)]\,dy$

$A = \int_a^b [0 - k(y)]\,dy$ since y-axis is $h(y) = 0$

$A = \int_a^b -k(y)\,dy$

This can also be written as $A = \left|\int_a^b k(y)\,dy\right|$.