Simultaneous Equations

Types of Questions

A **Solve** the given simultaneous linear and non-linear equations.

B **Find the**
- **(i) coordinates of the points of intersection,**
- **(ii) distance between the points of intersection,**
- **(iii) midpoint of the points of intersection,**

given equations of a line and a curve.

Worked Examples

Type A Examples

Type A Example 1

Question Solve the simultaneous equations $2x + y = 7$ and $3x^2 + y^2 = 28$.

Given Simultaneous linear equation $2x + y = 7$ and non-linear equation $3x^2 + y^2 = 28$

Required Solve the simultaneous equations.

Steps

1 For the linear equation, express one variable in terms of the other variable, i.e. make one variable (preferably with coefficient of one) the subject.

$2x + y = 7$

$y = -2x + 7$ ------(1)

2 Multiply the non-linear equation by LCM of denominators to eliminate denominators for the non-linear equation.

Not applicable as there are no fractions.

3 Substitute the linear equation into the non-linear equation.

$3x^2 + y^2 = 28$ ------(2)

Sub (1) into (2),

$3x^2 + (-2x + 7)^2 = 28$

4 Arrange in $ax^2 + bx + c = 0$ form.

$3x^2 + 4x^2 - 28x + 49 - 28 = 0$

$7x^2 - 28x + 21 = 0$

$x^2 - 4x + 3 = 0$

5 Solve by factorising or using quadratic formula.

$(x - 1)(x - 3) = 0$

$\boldsymbol{x = 1}$ **or** $\boldsymbol{x = 3}$

6 Substitute values of x into the linear equation to find the corresponding values of y or vice-versa.

Sub $x = 1$ into (1), $y = -2(1) + 7$

$\boldsymbol{y = 5}$

Sub $x = 3$ into (1), $y = -2(3) + 7$

$\boldsymbol{y = 1}$

Type A Example 2

Question Solve the simultaneous equations $2y + 5x = 1$ and $\frac{y}{x} + 2 = \frac{2x}{y} + \frac{1}{xy}$.

Given Simultaneous linear equation $2y + 5x = 1$ and non-linear equation $\frac{y}{x} + 2 = \frac{2x}{y} + \frac{1}{xy}$

Required Solve the simultaneous equations.

Steps

1 For the linear equation, express one variable in terms of the other variable, i.e. make one variable (preferably with coefficient of one) the subject.

$2y + 5x = 1$

$$x = \frac{1 - 2y}{5} \qquad \text{------(1)}$$

2 Multiply the non-linear equation by LCM of denominators to eliminate denominators for the non-linear equation.

$$\frac{y}{x} + 2 = \frac{2x}{y} + \frac{1}{xy}$$

$y^2 + 2xy = 2x^2 + 1$ ------(2)

3 Substitute the linear equation into the non-linear equation.

Sub (1) into (2),

$$y^2 + 2y\left(\frac{1 - 2y}{5}\right) = 2\left(\frac{1 - 2y}{5}\right)^2 + 1$$

4 Arrange in $ax^2 + bx + c = 0$ form.

$$y^2 + \left(\frac{2y - 4y^2}{5}\right) = \frac{2(1 - 4y + 4y^2)}{25} + 1$$

$25y^2 + 5(2y - 4y^2) = 2 - 8y + 8y^2 + 25$

$25y^2 + 10y - 20y^2 = 2 - 8y + 8y^2 + 25$

$-3y^2 + 18y - 27 = 0$

$y^2 - 6y + 9 = 0$

5 Solve by factorising or using quadratic formula.

$(y - 3)(y - 3) = 0$

$\boldsymbol{y = 3}$

6 Substitute value(s) of y into the linear equation to find the corresponding value(s) of x or vice-versa.

Sub $y = 3$ into (1), $x = \frac{1 - 2(3)}{5}$

$\boldsymbol{x = -1}$

Type B Example

Type B Example

Question The line $y - 2x = 1$ intersects the curve $x^2 + y^2 = xy + 7$ at the points A and B.
Find (i) the coordinates of A and B,
(ii) the distance AB,
(iii) the midpoint of the line AB.

Given Simultaneous linear equation $y - 2x = 1$ and non-linear equation $x^2 + y^2 = xy + 7$

Required Find (i) the coordinates of A and B,
(ii) the distance AB,
(iii) the midpoint of the line AB.

Steps

1 For the linear equation, express one variable in terms of the other variable, i.e. make one variable (preferably with coefficient of one) the subject.

$y - 2x = 1$

$y = 2x + 1$ ------(1)

2 Substitute the linear equation into the non-linear equation.

$x^2 + y^2 = xy + 7$ ------(2)

Sub (1) into (2),

$x^2 + (2x + 1)^2 = x(2x + 1) + 7$

3 Arrange in $ax^2 + bx + c = 0$ form.

$x^2 + 4x^2 + 4x + 1 = 2x^2 + x + 7$

$3x^2 + 3x - 6 = 0$

$x^2 + x - 2 = 0$

4 Solve by factorising or using quadratic formula.

$(x + 2)(x - 1) = 0$

$x = -2$ or $x = 1$

5 Substitute values of x into the linear equation to find the corresponding values of y or vice-versa and express the answers in coordinate form (x, y)

Sub $x = -2$ into (1), $y = 2(-2) + 1$

$= -3$

Sub $x = 1$ into (1), $y = 2(1) + 1$

$= 3$

The coordinates of A and B are $(-2, -3)$ and $(1, 3)$ respectively.

6 Find the distance between the points of intersection.

$AB = \sqrt{[3 - (-3)]^2 + [1 - (-2)]^2}$

$AB = \sqrt{36 + 9}$

$= \sqrt{45}$

$= \mathbf{3\sqrt{5}}$ **units**

7 Find the midpoint of the points of intersection.

$\text{Midpoint} = \left(\frac{1 - 2}{2}, \frac{3 - 3}{2}\right)$

$= \left(\mathbf{-\frac{1}{2}, 0}\right)$