Solutions to Quadratic Equations

Solving Quadratic Equations by Factorisation

(i) Using "cross method" to factorise a quadratic expression:

$2x^2 + 5x - 3 = (x + 3)(2x - 1)$

x	$+3$	$+6x$
$2x$	-1	$-x$
$2x^2$	-3	$+5x$

(ii) Using factorisation to solve a quadratic equation:

$$2x^2 + 5x = 3$$
$$(x + 3)(2x - 1) = 0$$
$$x + 3 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$x = -3 \qquad\qquad x = \frac{1}{2}$$

Example 1

Solve $6x^2 + 11x - 7 = 0$ by factorisation.

Solution:

$$6x^2 + 11x - 7 = 0$$
$$(2x - 1)(3x + 7) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad 3x + 7 = 0$$
$$x = \frac{1}{2} \qquad\qquad x = -2\frac{1}{3}$$

Example 2

(a) Factorise $3x^2 + 5x - 2$.

(b) Hence, solve $3x^2 + 5x - 2 = 0$.

Solution: (a) $3x^2 + 5x - 2 = (3x - 1)(x + 2)$

(b)
$$3x^2 + 5x - 2 = 0$$
$$(3x - 1)(x + 2) = 0$$
$$3x - 1 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = \frac{1}{3} \qquad\qquad x = -2$$

Solving Quadratic Equations by Completing the Square

A quadratic equation may be written in the form of a complete square $(x + h)^2 = k$, where h and k are real numbers.

Example 3

Solve $(x - 4)^2 = 25$.

Solution:
$$
\begin{aligned}
(x - 4)^2 &= 25 \\
x - 4 &= \pm 5 \\
x &= 4 \pm 5 \\
x &= -1, 9
\end{aligned}
$$

Example 4

Solve $(3x - 1)^2 - 8 = 0$ by completing the square. Give your answers correct to 2 decimal places.

Solution:
$$
\begin{aligned}
(3x - 1)^2 - 8 &= 0 \\
(3x - 1)^2 &= 8 \\
3x - 1 &= \pm \sqrt{8} \\
3x &= 1 \pm \sqrt{8} \\
x &= \frac{1 \pm \sqrt{8}}{3} \\
&= -0.61, 1.28 \quad \text{(2 d.p.)}
\end{aligned}
$$

General Solution to Quadratic Equations

A quadratic equation of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$, can be solved using the general formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 5

Solve $x^2 - 7 = 5x$, giving your answers correct to 2 decimal places.

Solution:
$$
\begin{aligned}
x^2 - 7 &= 5x \\
x^2 - 5x - 7 &= 0 \\
a = 1,\ b &= -5,\ c = -7 \\
x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-7)}}{2(1)} \\
&= \frac{5 \pm \sqrt{53}}{2} \\
&= -1.14 \text{ or } 6.14 \quad \text{(2 d.p.)}
\end{aligned}
$$

Note

The nature of roots of a quadratic equation depends on the value of its discriminant $b^2 - 4ac$.

(i)	*If*	*$b^2 - 4ac$ is positive,*	*then there are two real and distinct roots.*
(ii)	*If*	*$b^2 - 4ac$ is zero,*	*then there are two real and equal roots.*
(iii)	*If*	*$b^2 - 4ac$ is negative,*	*then there are no real roots; the roots are imaginary.*

1.4 Solving Fractional Equations that are Reducible to Quadratic Equations

A fractional equation that can be reduced and expressed as $ax^2 + bx + c = 0$, where $a \neq 0$, can be solved using the methods in section **1.1**, **1.2** or **1.3**.

Example 6

Solve $\frac{1}{x-1} - \frac{1}{x} = 5$, giving your answers correct to 3 significant figures.

Solution:

$$\frac{1}{x-1} - \frac{1}{x} = 5$$

$$\frac{x-(x-1)}{x(x-1)} = 5$$

$$\frac{1}{x(x-1)} = 5$$

$$1 = 5x(x-1)$$

$$5x^2 - 5x - 1 = 0$$

Using general formula: $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(5)(-1)}}{2(5)}$

$$= \frac{5 \pm \sqrt{45}}{10}$$

$$= -0.171, 1.17 \quad \text{(3 sig. fig.)}$$