Linear Inequalities

Inequalities

Rules

(i) If $x > y$ and $y > z$ $\Rightarrow$ $x > z$

(ii) If $x > y$ $\Rightarrow$ $x + z > y + z$ and $x - z > y - z$

(iii) If $x > y$ and $z > 0$ $\Rightarrow$ $xz > yz$ and $\frac{x}{z} > \frac{y}{z}$

(iv) If $x > y$ and $z < 0$ $\Rightarrow$ $xz < yz$ and $\frac{x}{z} < \frac{y}{z}$

Note

Change the inequality sign when both sides of an inequality are multiplied or divided by a negative number.

Example 1

Using the correct inequality symbols, fill in the blanks to make the following statements true.

(a) If $8 < 1$ and $1 < m$, then $8 \;\square\; m$.

(b) If $3p > 5q$, then $3p - 7 \;\square\; 5q - 7$.

(c) If $16 \geqslant 7$ and $n > 0$, then $\frac{16}{n} \;\square\; \frac{7}{n}$.

(d) If $r < 4$, then $-2r \;\square\; -8$.

Solution:

(a) Using Rule (i): If $8 < 1$ and $1 < m$,
then $8 < m$. (Keep the inequality sign)

(b) Using Rule (ii): If $3p > 5q$,
then $3p - 7 > 5q - 7$. (Keep the inequality sign)

(c) Using Rule (iii): If $16 \geqslant 7$ and $n > 0$,
then $\frac{16}{n} \geqslant \frac{7}{n}$. (Keep the inequality sign)

(d) Using Rule (iv): If $r < 4$,
then $-2r > -8$. (Invert the inequality sign)

Example 2

Solve the following linear inequalities.

(a) $d - 2 \geqslant 5$

(b) $1 - g < 3$

(c) $4h \leqslant 6 - h$

(d) $\frac{k}{2} - 3 > 0$

(e) $5(1 - m) < 2(3m + 2)$

(f) $\frac{2n-1}{3} \geqslant \frac{n+7}{4}$

Solution:

(a)
$$\begin{aligned} d - 2 &\geqslant 5 \\ d &\geqslant 5 + 2 && \text{(both sides + 2)} \\ d &\geqslant 7 \end{aligned}$$

(b)
$$\begin{aligned} 1 - g &< 3 \\ -g &< 3 - 1 && \text{(both sides – 1)} \\ -g &< 2 \\ g &> -2 && \text{(both sides multiplied by – 1, invert the inequality sign)} \end{aligned}$$

(c)
$$\begin{aligned} 4h &\leqslant 6 - h \\ 4h + h &\leqslant 6 && \text{(both sides + } h\text{)} \\ 5h &\leqslant 6 \\ h &\leqslant 1\tfrac{1}{5} && \text{(both sides divided by 5)} \end{aligned}$$

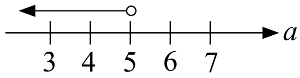
(d)
$$\begin{aligned} \frac{k}{2} - 3 &> 0 \\ \frac{k}{2} &> 3 && \text{(both sides + 3)} \\ k &> 6 && \text{(both sides multiplied by 2)} \end{aligned}$$

(e)
$$\begin{aligned} 5(1 - m) &< 2(3m + 2) \\ 5 - 5m &< 6m + 4 \\ 5 - 5m - 6m &< 4 && \text{(both sides – } 6m\text{)} \\ 5 - 11m &< 4 \\ -11m &< -1 && \text{(both sides – 5)} \\ m &> \frac{1}{11} && \text{(both sides divided by – 11, invert the inequality sign)} \end{aligned}$$

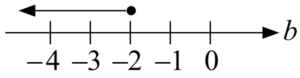
(f)
$$\begin{aligned} \frac{2n-1}{3} &\geqslant \frac{n+7}{4} \\ 4(2n - 1) &\geqslant 3(n + 7) \\ 8n - 4 &\geqslant 3n + 21 \\ 8n - 4 - 3n &\geqslant 21 && \text{(both sides – } 3n\text{)} \\ 5n - 4 &\geqslant 21 \\ 5n &\geqslant 21 + 4 && \text{(both sides + 4)} \\ 5n &\geqslant 25 \\ n &\geqslant 5 && \text{(both sides divided by 5)} \end{aligned}$$

3.2 Representing Inequalities Using a Number Line

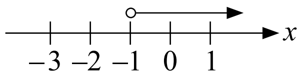
(i) $a < 5$



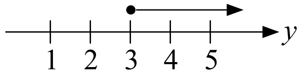
(ii) $b \leqslant -2$



(iii) $x > -1$



(iv) $y \geqslant 3$



Note

1. *Use '○' (circle) to represent '<' and '>'. It means that the value on the number line* *__is not included__.*
2. *Use '●' (circle-dot) to represent '⩽' and '⩾'. It means that the value on the number line* *__is included__.*

Example 3

(a) Solve the inequality $\frac{2x}{3} < 8 - \frac{4x}{9}$. Illustrate your answer using a number line.

(b) Write down the greatest integer value of x that satisfies $\frac{2x}{3} < 8 - \frac{4x}{9}$.

Solution: (a)

$$\frac{2x}{3} < 8 - \frac{4x}{9}$$

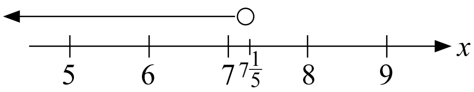
$$\frac{2x}{3} + \frac{4x}{9} < 8$$

$$\frac{3(2x) + 4x}{9} < 8$$

$$\frac{10x}{9} < 8$$

$$10x < 72$$

$$x < 7\frac{1}{5}$$



(b) Since $x < 7\frac{1}{5}$, the greatest integer value of x is 7.

Example 4

Solve the following linear inequalities and illustrate each of the solutions using a number line.

(a) $6x - 15 < 3x - 9$ and $2x + 13 \geqslant 7$

(b) $x + 3 \leqslant 4x + 2 < 2x + 18$

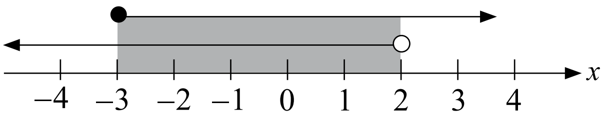
Solution: (a)

$6x - 15 < 3x - 9$ and $2x + 13 \geqslant 7$

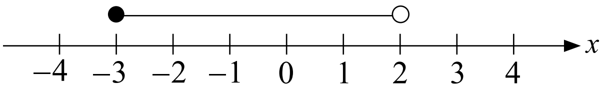
$6x - 3x < -9 + 15$ $\qquad 2x \geqslant 7 - 13$

$3x < 6$ $\qquad 2x \geqslant -6$

$x < 2$ $\qquad x \geqslant -3$



The solution is represented by the shaded region.
Hence, the solution is $-3 \leqslant x < 2$.
The solution illustrated on a number line is shown below.



(b) $x + 3 \leqslant 4x + 2 < 2x + 18$

We have

$x + 3 \leqslant 4x + 2$ and $4x + 2 < 2x + 18$

$x - 4x \leqslant 2 - 3$ $\qquad 4x - 2x < 18 - 2$

$-3x \leqslant -1$ $\qquad 2x < 16$

$x \geqslant \frac{1}{3}$ $\qquad x < 8$

Hence, the solution is $\frac{1}{3} \leqslant x < 8$.