Volume and Surface Area of Prisms and Cylinders

Concept of Volume and Conversion of Units of Volume

(i) Concept of Volume

The volume of an object is the **amount** of space occupied by the object.
The volume of a solid or a fluid is measured using units such as **millilitres** (ml), **litres** (l), **kilolitres** (kl), cubic centimetres (cm^3) and cubic metres (m^3).

(ii) Conversion of Units of Volume

$$1 \text{ m}l = 1 \text{ cm}^3 = \frac{1}{1000}\ l$$

$$1\ l = 1000 \text{ m}l = 1000 \text{ cm}^3$$

$$1 \text{ k}l = 1000\ l = 1\ 000\ 000 \text{ m}l = 1 \text{ m}^3$$

Example 1

Convert each of the following to the required unit:

(a) $1.8\ l$ = __________ mm^3

(b) $\frac{4}{25}$ kl = __________ m^3

(c) 54 000 ml = __________ m^3

(d) 0.021 m^3 = __________ l

(e) $1\frac{3}{4}\ l$ = __________ cm^3

(f) 245 000 cm^3 = __________ kl

Solution:

(a)
$$\begin{aligned} 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} &= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} \\ 1 \text{ cm}^3 &= 1000 \text{ mm}^3 \\ 1800 \text{ cm}^3 &= 1\ 800\ 000 \text{ mm}^3 \\ 1.8\ l &= 1\ 800\ 000 \text{ mm}^3 \end{aligned}$$

(b)
$$\begin{aligned} 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} &= \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m} \times \frac{1}{100} \text{ m} \\ 1 \text{ cm}^3 &= \frac{1}{1\ 000\ 000} \text{ m}^3 \\ 1\ 000\ 000 \text{ cm}^3 &= 1 \text{ m}^3 \\ 1000\ l &= 1 \text{ m}^3 \\ 1 \text{ k}l &= 1 \text{ m}^3 \\ \frac{4}{25} \text{ k}l &= \frac{4}{25} \text{ m}^3 \end{aligned}$$

(c) $1\text{ cm}^3 = \frac{1}{1\,000\,000}\text{ m}^3$

$1\text{ m}l = \frac{1}{1\,000\,000}\text{ m}^3$

$54\,000\text{ m}l = \frac{54\,000}{1\,000\,000}\text{ m}^3$

$= 0.054\text{ m}^3$

(d) $1\text{ m}^3 = 1000\ l$

$0.021\text{ m}^3 = 0.021 \times 1000\ l$

$= 21\ l$

(e) $1\ l = 1000\text{ m}l$

$= 1000\text{ cm}^3$

$1\frac{3}{4}\ l = 1.75 \times 1000\text{ cm}^3$

$= 1750\text{ cm}^3$

(f) $1\text{ m}l = \frac{1}{1\,000\,000}\text{ k}l$

$1\text{ cm}^3 = \frac{1}{1\,000\,000}\text{ k}l$

$245\,000\text{ cm}^3 = 245\,000 \times \frac{1}{1\,000\,000}\text{ k}l$

$= 0.245\text{ k}l$

Volume and Surface Area of Cubes and Cuboids – Revision

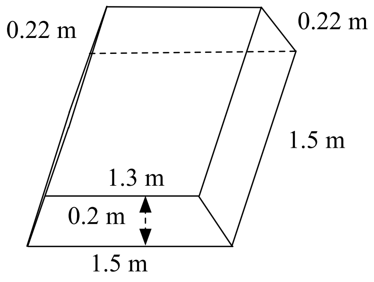
	Figure	***Volume*** (V)	***Surface Area*** (S.A.)
(i)	**Cube**	$V = \text{length}^3$ $= L^3$	$S.A. = 6 \times \text{length}^2$ $= 6L^2$
(ii)	**Cuboid**	$V = \text{length} \times \text{width} \times \text{height}$ $= L \times W \times H$	$S.A. = 2(\text{length} \times \text{width} + \text{length} \times \text{height} + \text{width} \times \text{height})$ $= 2(L \times W + L \times H + W \times H)$

Volume and Surface Area of Prisms and Cylinders

	Figure	***Volume*** (V)	***Surface Area*** (S.A.)
(i)	**Prism**	$V = \text{base area} \times \text{height}$	$S.A. = \text{perimeter of base} \times \text{height} + 2(\text{base area})$
(ii)	**Cylinder**	$V = \text{base area} \times \text{height}$ $= \pi r^2 h$	$S.A. = \text{curved surface area} + 2(\text{area of the base circle})$ $= 2\pi r(h + r)$

Example 2

An open tray with a square base is 1.5 m long and 0.2 m deep.



Find
(a) its volume,
(b) its surface area.

Solution: (a) The tray is a right prism.

Area of the trapezium $= \frac{1}{2} \times (1.3 + 1.5) \times 0.2$

$= 0.28 \text{ m}^2$

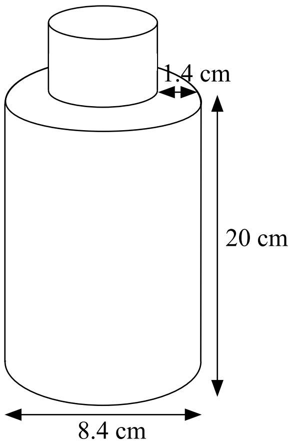
Volume of the tray = volume of the prism
= area of the trapezium × breadth
$= 0.28 \times 1.5$
$= 0.42 \text{ m}^3$

(b) Surface area of the tray
= 2 × (area of the trapezium + area of the rectangular face) + area of the square base
$= 2 \times (0.28 + 0.22 \times 1.5) + 1.5 \times 1.5$
$= 2 \times 0.61 + 2.25$
$= 3.47 \text{ m}^2$

Example 3

A container consists of two cylinders, one of thickness 1.4 cm and one of negligible thickness. The air space of the container is 616 cm^3. Find
(a) the height of the container,
(b) the total surface area of the container if it is opened on the top.

[Take $\pi = 3\frac{1}{7}$]



Solution: (a) Internal radius $= \frac{8.4}{2} - 1.4$

$= 4.2 - 1.4$

$= 2.8$ cm

Volume of the air space = internal volume of the container

$= \pi \times \text{radius}^2 \times \text{height}$

$$\frac{22}{7} \times 2.8^2 \times \text{height} = 616 \text{ cm}^3$$

$$\text{Height} = \frac{616}{\frac{22}{7} \times 2.8^2}$$

$$= 25 \text{ cm}$$

(b) Base area $= \pi \times \text{radius}^2$

$= \frac{22}{7} \times 4.2^2$

$= 55.44 \text{ cm}^2$

Area of the circle on top $= \pi r^2$

$= \frac{22}{7} \times 2.8^2$

$= 24.64 \text{ cm}^2$

Total curved surface area of the bigger cylinder (inner and outer)

$= 2 \times \frac{22}{7} \times 4.2 \times 20 + 2 \times \frac{22}{7} \times 2.8 \times 20$

$= 528 + 352$

$= 880 \text{ cm}^2$

Total curved surface area of the smaller cylinder (inner and outer)

$= 2 \times 2 \times \frac{22}{7} \times 2.8 \times (25 - 20)$

$= 176 \text{ cm}^2$

Total surface area = 2(base area) − area of the circle on top + total curved surface area of the bigger cylinder + total curved surface area of the smaller cylinder

$= 2(55.44) - 24.64 + 880 + 176$

$= 1142.24$

$= 1140 \text{ cm}^2$ (3 s.f.)