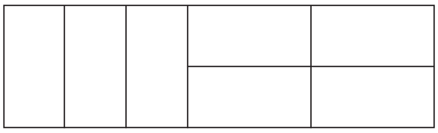
1. Seven identical dominoes of size 1 cm by 2 cm and with identical faces on both sides are arranged to cover a rectangle of size 2 cm $\times$ 7 cm. One possible arrangement is shown below. Find the total number of ways in which the rectangle can be covered by the seven dominoes.



2. Among 64 students, 28 of them like Science, 41 like Mathematics and 31 like English. 18 of them like both Mathematics and English. 15 students like both Science and English. 11 students like both Science and Mathematics. How many students like all the three subjects?

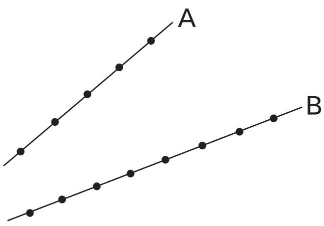
3. What is the value of the digit in the ones place of the following?

$1 \times 3 \times 5 \times 7 \times 9 \times 11 \times 13 \times \ldots \times 2017 \times 2019$

4. Evaluate

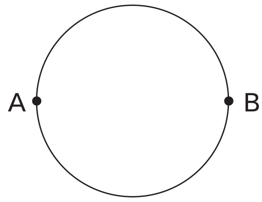
$$\frac{1+2+3+4+5+6+7+6+5+4+3+2+1}{7777777 \times 7777777}.$$

5. There are 5 dots on line A and 8 dots on line B.



Find the total number of triangles that can be formed using any 3 dots as their vertices.

6. Jonathan and Cindy run on a circular track where AB is the diameter of the track, as shown below.



If Jonathan and Cindy run towards each other at the same time from Point A and Point B respectively, it will take them 40 seconds before they meet. If they start running at the same time but in the same direction, it will take Jonathan 280 seconds to catch up with Cindy. What is the ratio of their speeds?

7. Find the value of

$1^2 - 2^2 + 3^2 - 4^2 + \cdots + \quad + 2017^2 - 2018^2$.

8. Between 1 and 2018, how many numbers are multiples of 5 or 7?

9. The sum of the digits of a 3-digit number is 18. The tens digit is 1 more than the ones digit. If the hundreds digit and the ones digits are swapped, the difference between the new number and the original number is 396.
What is the original number?

69. The last 4 digits of the product of $725 \times 835 \times 932 \times m$ is 0000. Find the smallest value of m.

70. In the number pattern below, find the 6th and 7th terms.

6, 10, 17, 27, 40, (   ), (   ), ...

71. Find the value of the digit represented by each letter, given that each letter must represent a different digit.

(a)

$$\begin{array}{r} A\;B\;C\;A\;B \\ \times \qquad\qquad D \\ \hline E\;E\;E\;E\;E\;E \\ \hline \end{array}$$

(b)

$$\begin{array}{r} a\;b\;c\;d \\ \times \qquad\quad 4 \\ \hline d\;c\;b\;a \\ \hline \end{array}$$

72. 2009 rescue parcels are distributed to 11 regions affected by the earthquake. The order of distribution is shown below.

Region 1, Region 3, Region 6, Region 8, Region 11, Region 2, Region 5, Region 7, ...

Which region will get the last parcel?

73. A car travelled towards Town B from Town A. A van headed to Town A from Town B at the same time. They met each other 80 km from the midway of the two towns. The speed of the car was 1.4 times that of the van. Find the distance between the two towns.

74. There are four Level Six classes in a school. The number of students in Class 6B, 6C and 6D is 122. The number of students in Class 6A, 6B and 6C is 123. The number of students in Class 6B and Class 6C is 2 less than the number of students in Class 6A and Class 6D. What is the total number of students in Level Six?

75. A farmer has a certain number of eggs. If he counts the eggs in groups of 3, there will be a remaining of 2 eggs. If he counts the eggs in groups of 4, 3 eggs will remain. The remainders will be 5 and 6 respectively if the farmer counts the eggs in groups of 6 and 7 respectively. How many eggs, at least, does the farmer have?

76. A basketball is passed among Melvin, Nigel, Owen and Patrick for a total of 4 times. It starts with Melvin and ends with him as well. How many ways are there to pass the ball?

77. What is the value of the digit represented by each letter?

(a)

$$\begin{array}{r} A\;B \\ \times \; B\;A \\ \hline 6\;7\;2 \\ 3\;3\;6\; \\ \hline 4\;0\;3\;2 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} A\;B\;C\;D \\ -\; D\;A\;B\;C \\ \hline 2\;9\;2\;5 \\ \hline \end{array}$$

85. Car A and Car B left Town A for Town B at 60 km/h and 70 km/h respectively. At the same time, Car C left Town B for Town A at 80 km/h. Car C passed Car A 1.5 hours after it passed Car B. Find the distance between Town A and Town B.

86. 1 US Dollar (USD) can be exchanged for 15 000 Vietnam Dong (VD) on a particular day. 80 USD can be exchanged for 120 Singapore Dollars (S$) on the same day. How much S$ can 20000 VD be exchanged for on that day?

87. When the three numbers 1238, 1596 and 2491 are divided by a positive integer *m*, the remainders are all equal to n. Find the value of $(m + n)$.

88. Two glasses contain 140 ml of milk each. Half of the milk in the first glass is poured into the second glass. After that, $\frac{1}{3}$ of the milk in the second glass is poured back into the first glass. $\frac{1}{4}$ of milk in the first glass is then poured into the second glass again, and so on. What is the volume of milk in each glass after pouring 100 times?

89. The ones digit of a 6-digit number is 7. The product of this 6-digit number by 5 is a 6-digit number that begins with 7. The rest of the 5 digits of the product equal the first 5 digits of the original number. Find the 6-digit number.

90. Find the sum of all the digits in $\underbrace{111 \ \ldots \ 111}_{207\ 1\text{'s}} \times \underbrace{111 \ \ldots \ 111}_{207\ 1\text{'s}}$.

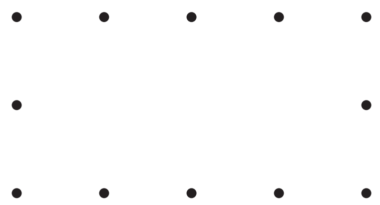
91. A car left Town A for Town B. At the same time, a truck left Town B for Town A. They passed each other 80 km from Town A. Each of them made a U-turn upon reaching its destination and travelled in the opposite direction. They then passed each other again 34 km from Town B. How far was Town B from Town A?

92. A triathlete walks for 1 km, runs for the next 2 km and cycles for the last 3 km. He can cycle twice as fast as he runs and runs twice as fast as he walks. The total time for the three activities is 12.5 minutes more than if he has cycled for the whole journey. How much time does he take for each activity?

93. A string of 3 digits is written after the number 1992 to form a 7-digit number, which can be divided by 2, 3, 5 and 11. Find the smallest possible value of the 7-digit number.

94. In a rush to finish the last question on addition, little Kevin miswrote the ones digit "5" as "6". He further miswrote the tens digit "8" as "3". As a result, he got an answer of 234. What should the answer be?

131. In the figure below, a rectangle is formed using 12 thumbtacks. Every two thumbtacks are 10 cm apart.



A piece of string is used to form triangles of area 300 cm^2 using any three points. Find the number of such triangles.

132. Lucas told his friend: "The product of my ranking, my score based on a full mark of 100, and my age in the competition is 2280."

(a) How old was Lucas?

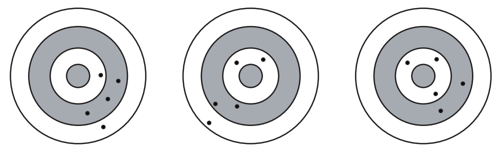
(b) What was his ranking in the competition?

133. James needs to purchase 4 \$1-stamps. He tells the cashier that he wants the 4 stamps to be joined together. In how many ways can the cashier deliver this special request from the stamps in the figure below?

\$1	\$1	
\$1	\$1	\$1
\$1	\$1	\$1

134. What are the last two digits of 2019^{2019}?

135. Three policemen were honing their shooting skills at a firing range. Their score boards were then sent for analysis.



Matthew: **I have the highest score!**
I have 20 points more than Leon.
My score is 320.

Jason: **I am the marksman of the day.**
I am 40 points ahead of Leon.
Leon has 20 points more than Matthew.

Leon: **Aghh! I have to better my 300 points.**
Matthew has good potential.
Jason's score of 360 points is remarkable!

One of the statements made by the policemen was false. Find their actual scores.

136. The ratio of a father's age to his son's age is 5 : 1. The ratio becomes 2 : 1 fifteen years later. How old is each of them now?

137. Uncle Philip bought 1000 shares of an IT company. Shortly, the share price dropped by 20%. Find the percentage increase that must occur before he can sell the shares to break even.

138. The number of black beads is $\frac{1}{7}$ more than the number of white beads. It is, however, $\frac{1}{5}$ less than the number of red beads. What is the ratio of the number of white beads to the number of black beads to the number of red beads?

214. Find the value of $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \cdots + 99^2 - 100^2$.
[Hint: $a^2 - b^2 = (a + b)(a - b)$]

215. There are 64 tennis balls but, only 18 tubes. Each tube can hold at most 6 balls. How many tubes, at least, have the same number of balls?

216. What is the 6th term in the number sequence below?

1, 11, 21, 1211, 111221, 312211, ...

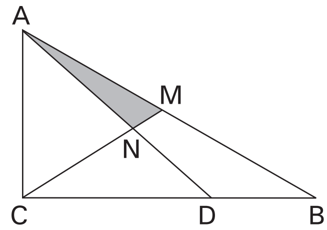
217. There are four boys and two girls, each of different age:

The youngest child is 6 years old.
The eldest boy is 4 years older than the youngest girl.
The eldest girl is older than the youngest boy by 4 years too.
The eldest child is 12 years old.

How old is the eldest girl?

218. The remainder is 5 when 200 is divided by a whole number. The remainders are 1 and 10 respectively if 300 and 400 respectively are divided by the same whole number. What is the value of the whole number?

219. ABC is a right-angled triangle.
Given AC = 8, CD = 8, BC = 12, AM = BM.
Find the area of the shaded region.



220. Study the following pattern.

$1 = 1 = 1^2$
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
$1 + 3 + 5 + 7 + 9 = 25 = 5^2$
$\vdots$

Given that $1 + 3 + 5 + 7 + \cdots + 2017 + 2019 = k^2$, where k is a positive integer, find the value of k.

221. 500 books are to be distributed to a certain number of libraries. Each library, however, must not get more than 15 books. How many libraries, at least, will get the same number of books?

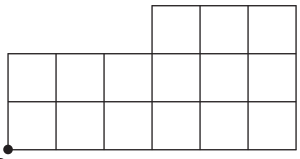
222. Find the 8th term in the number sequence below.

102, 105, 111, 114, 120, 123, ...

255. A 3-digit number is 25 times the sum of the three digits. What is the greatest possible value of this 3-digit number?

256. Evaluate $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72}$.

257. The figure below shows a piece of land that is made up of 13 identical squares. A land surveyor has to divide it into two equal halves. On the figure below, draw a straight line from P to achieve this.



258. A rectangle is formed by reducing one side of a square by 30% and increasing the other side by 3 m. The area of the rectangle remains the same as the square. What is the area of the square?

259. A truck delivers goods from Town A to Town B. It can travel 90 km/h when it is empty but only at 60 km/h when it is loaded with goods. Find the distance between the two towns if it can cover the 6 trips in 12 hours.

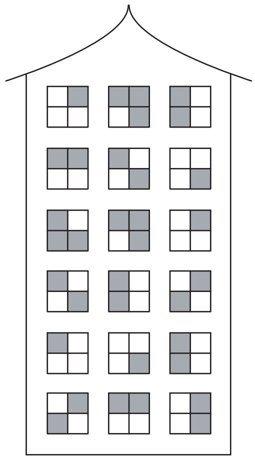
260. A DVD player was priced at 140% of its cost price. It was then sold at 90% of its selling price. A gift voucher worth $50 was given to the customer at the same time. The profit earned was $158 as a result. What was the cost price of the DVD player?

261. Order the fractions below, from the largest to the smallest.

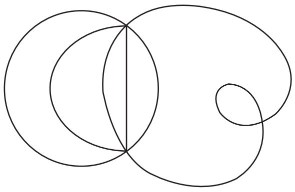
$$\frac{135}{428}, \quad \frac{200}{688}, \quad \frac{268}{960}$$

262. Mark can harvest crops in 9 days. His father needs 15 days to complete the same task. If Mark works on the first day, his father works on the second day, Mark works again on the third day, his father works on the fourth day, and so on, how many days does it take to harvest $\frac{4}{5}$ of the crops?

263. Each floor of a 6-storey apartment has 3 window frames. Each frame consists of 4 pieces of glass, which are either blue ■ or white □. Each frame also represents one number. The six sets of numbers representing each floor are 167, 451, 205, 983, 385 and 640. Find the number that represents the fourth floor if the first floor is represented by 167.



264. A part of a botanic garden has a walking path as shown below.

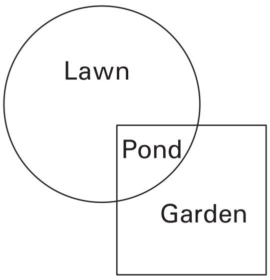


Mark the position of a gate where a visitor can enter and walk the path exactly once.

265. A cargo leaves Port A for Port B every day. The whole journey takes 8 days. Another cargo leaves Port B for Port A every day at exactly the same time, but needs only 7 days to arrive. How many cargoes from Port B will meet a cargo from Port A during the whole journey?

266. Two candles have different heights. One takes 4 hours to burn completely while the other takes only 3 hours to close. At what time must the candles be lit so that the ratio of their heights is 2 : 1 at 2.00 pm?

267. An architect designs a park as shown below. $\frac{7}{9}$ of the circle is the lawn. $\frac{3}{4}$ of the square is the garden. The overlap section is a pond. What is the area of the pond if the area of the lawn is 200 m^2 greater than that of the garden?



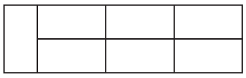
268. Find the remainder when the 2011th term of the sequence below is divided by 9.
1, 2, 6, 16, 44, 120, 328, 896, ...

269. A publishing house in Singapore has to deliver 2000 and 1600 Mathematics Olympiad books to Korea and the Philippines respectively. Its branch office in Hong Kong is able to deliver 2400 copies only. The shipment costs \$500 per 200 copies to the Philippines and \$300 per 200 copies to Korea. The Singapore office takes care of the remaining copies and it costs \$800 to deliver every 200 copies to Korea and \$400 to deliver every 200 copies to the Philippines. What is the least cost to deliver the books?

270. Three cards carry a number each. They are distributed at random to three players where the number is then recorded. This is repeated a few times and the total scores obtained by the players are 22, 20, 18 respectively.
What is the number on each card?

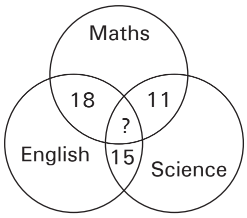
1. It suffices to consider four scenarios: 1 vertical, 3 vertical, 5 vertical and 7 vertical.

1 vertical: example
4 ways



3 vertical: 10 ways
5 vertical: 6 ways
7 vertical: 1 way
Ans: 21 ways

2.



$28 + 41 + 31 = 100$
$18 + 15 + 11 = 44$ students are counted twice.
$100 - 44 = 56$ students like either 1 or 2 subjects.
Number of students $= 64$
$64 - 56 =$ **8 students** like all the three subjects.

3. It is sufficient to look only at the product of 1, 3, 5, 7 and 9.
$1 \times 3 \times 5 \times 7 \times 9 = 945$, where the digit in the ones place is 5.
Hence, we have $5 \times 5 \times \ldots \times 5$.
The value of the digit in the ones place is **5**.

4. There are 7 pairs of 7 in $1 + 2 + \ldots + 7 + 6 + \ldots + 1$.
Therefore,

$$\frac{1 + 2 + \ldots + 7 + 6 + \ldots + 1}{7777777 \times 7777777}$$
$$= \frac{7 \times 7}{7777777 \times 7777777}$$
$$= \frac{1}{1111111 \times 1111111}$$
$$= \mathbf{\frac{1}{1234567654321}}$$

5. **Scenario 1: Using line A as base**

$${}^5C_2 = \frac{5 \times 4}{1 \times 2}$$
$$= 10$$

Total number of triangles, $10 \times {}^8C_1 = 80$.
Scenario 2: Using line B as base

$${}^8C_2 \times {}^5C_1 = \frac{8 \times 7}{1 \times 2} \times 5$$
$$= 28 \times 5$$
$$= 140$$

Ans: 220 ways

6. Both of them will cover half of a circumference if they run towards each other. Jonathan needs to cover half a circumference in order to catch up with Cindy if they run in the same direction.
Let the speed of Jonathon and Cindy be J and C respectively.
We can write their speeds as follows:
To meet up:
$(J + C) \times 40 =$ half a circumference
To catch up:
$(J - C) \times 280 =$ half a circumference
Equating the two statements, we have

$$(J + C) \times 40 = (J - C) \times 280$$
$$J + C = (J - C) \times 7$$
$$J + C = 7J - 7C$$
$$8C = 6J$$
$$J : C = 8 : 6$$
$$= 4 : 3$$

The ratio of Jonathan's speed to Cindy's speed is **4 : 3**.

7. Use the identity $a^2 - b^2 = (a - b)(a + b)$

$$= (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + \cdots + (2017 - 2018)(2017 + 2018)$$
$$= -(1 + 2 + 3 + 4 + \cdots + 2017 + 2018)$$
$$= -\frac{(1 + 2018) \times 2018}{2}$$
$$= \mathbf{-2\,037\,171}$$

8. $5 \times 7 = 35$
$2018 \div 5 = 403$ R 3
$2018 \div 7 = 288$ R 2
$2018 \div 35 = 57$ R 23
Number of multiples of 5 or 7
$= 403 + 288 - 57$
$=$ **634**

9. **Method 1:**
List down all the possible numbers in the table below.

Original number	New number	Difference
198	891	693
387	783	396 (✓)
576	675	99
765	567	198
954	459	495

Therefore, the original number is **387**.

Method 2:
Let the 3-digit number be $\overline{abc}$.
Hence, we have $\overline{cba} - \overline{abc} = 396$.

$$100c + 10b + a - 100a - 10b - c = 396$$
$$99c - 99a = 396$$
$$c - a = 396 \div 99 = 4$$
$$c = 4 + a \quad \text{----- (1)}$$
$$b - c = 1$$
$$c = b - 1 \quad \text{----- (2)}$$

Substitute (2) into (1):

$$b - a = 5$$
$$b = 5 + a \quad \text{----- (3)}$$
$$a + b + c = 18 \quad \text{----- (4)}$$

Substitute (1) and (3) into (4):

$$a + 5 + a + 4 + a = 18$$
$$3a + 9 = 18$$
$$3a = 9$$
$$a = 9 \div 3$$
$$= 3$$

Substitute $a = 3$ into (3): $b = 8$
Substitute $a = 3$ into (1): $c = 7$
The original number is **387**.

10. Let $\frac{1}{3} + \frac{1}{5} = a$ and $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} = b$.

Therefore, we have $(1 + a)(b) - (1 + b)(a)$

$= b + ab - a - ab$

$= b - a$

$= \frac{1}{3} + \frac{1}{5} + \frac{1}{7} - \frac{1}{3} - \frac{1}{5}$

$= \mathbf{\frac{1}{7}}$

11. (a) $^4P_3 = 4 \times 3 \times 2$

$= 24$

There are **24 ways** to arrange the three letters.

(b) $^{12}P_2 = 12 \times 11$

$= 132$

There are **132 ways** to do so.

12. Method 1:

Let the distance between Town A and Town B be a common multiple of 48 and 72, say 144.

Time taken to travel to Town B $= 144 \div 72$

$= 2$ h

Time taken to return to Town A $= 144 \div 48$

$= 3$ h

Total time taken $= 2 + 3$

$= 5$ h

Total distance travelled $= 144 \times 2$

$= 288$ km

Average speed $= \frac{288}{5} = \mathbf{57\frac{3}{5}}$ **km/h**

Method 2:

Ratio of speed $= 72 : 48 = 3 : 2$

Hence, ratio of time $= 2 : 3$

The time taken for the whole journey can be calculated as follows: $\frac{d}{2} + \frac{d}{3} = \frac{2d}{s}$ where s is the average speed in terms of units and d is the distance between Town A and Town B

$\frac{d}{2} + \frac{d}{3} = \frac{2d}{s}$

$\frac{5d}{6} = \frac{2d}{s}$

$5s = 12$

$s = 2\frac{2}{5}$ units

From the ratio of speed,

3 units $= 72$ km/h

1 unit $= 72 \div 3 = 24$ km/h

$2\frac{2}{5}$ units $= 2\frac{2}{5} \times 24 = 57\frac{3}{5}$ km/h

Therefore, average speed $= \mathbf{57\frac{3}{5}}$ **km/h**

13. Let the month of birth be m and the day of birth be d.

$31m + 12d = 213$

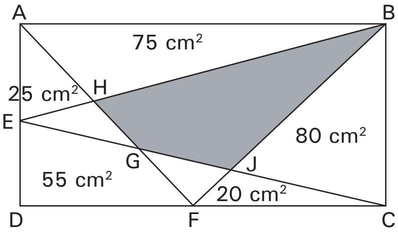
$31m = 213 - 12d$

$m = \frac{213 - 12d}{31}$

When $d = 10$, $m = \frac{213 - 120}{31} = 3$

His birthday is **March 10**.

14.



The area of △ABF is half the area of rectangle ABCD. The area of △BCE is also half the area of rectangle ABCD.

Hence, the sum of area of △ADF and △BCF is the area of △BCE.

Area of △BCE

= Area of shaded region + 80 + △EGH

Area △BCF + Area △ADF

= 80 + 20 + 55 + 25 + △EGH

= 180 + △EGH

Equate the 2 statements:

Area of shaded region + 80 + △EGH

= 180 + △EGH

Area of shaded region = 180 − 80

= **100 cm²**

15. $20172018 \times 20182017 - 20172017 \times 20182018$

Let $a = 20172017$

$b = 20182017$

$= (a + 1)b - a(b + 1)$

$= \cancel{ab} + b - \cancel{ab} - a$

$= b - a$

$= \mathbf{10\ 000}$

16. Write $29\frac{1}{2}$ as $\left(30 - \frac{1}{2}\right)$, $39\frac{1}{3}$ as $\left(40 - \frac{2}{3}\right)$ and $49\frac{1}{4}$ as $\left(50 - \frac{3}{4}\right)$.

$\left(30 - \frac{1}{2}\right) \times \frac{2}{3} + \left(40 - \frac{2}{3}\right) \times \frac{3}{4} + \left(50 - \frac{3}{4}\right) \times \frac{4}{5}$

$= 30 \times \frac{2}{3} - \frac{1}{2} \times \frac{2}{3} + 40 \times \frac{3}{4} - \frac{2}{3} \times \frac{3}{4}$

$+ 50 \times \frac{4}{5} - \frac{3}{4} \times \frac{4}{5}$

$= 20 - \frac{1}{3} + 30 - \frac{2}{4} + 40 - \frac{3}{5}$

$= 90 - \frac{1}{3} - \frac{1}{2} - \frac{3}{5}$

$= 90 - \frac{43}{30}$

$= \mathbf{88\frac{17}{30}}$

17. All 3-digit numbers that have the sum of the three digits equalling to 4 are as follows:

400, 112, 121, 211,
103, 301, 310, 130,
220, 202

There are altogether **10 numbers**.

18. If the car travels at 60 km/h, it has $60 \times 2 = 120$ km to go at 1.00 pm.

We can calculate the travelling time by treating it as a "catching up" problem.

Difference in speed $= 80 - 60 = 20$ km/h
Time taken when it travels at 80 km/h
$= 120 \div (80 - 20)$
$= 6$ h
Distance from Town A to Town B
$= 6 \times 80$
$= 480$ km
Time taken to arrive one hour later at 2.00 pm
$= 6 + 1$
$= 7$ h
Speed of the car $= 480 \div 7 = \mathbf{68\frac{4}{7}}$ **km/h**

19. Let the number of big boxes be m and the number of small boxes be n.

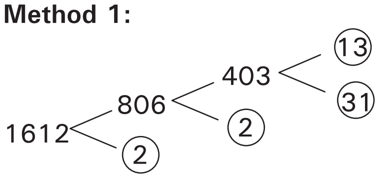
$$48m + 30n = 372$$
$$48m = 372 - 30n$$
$$m = \frac{372 - 30n}{48}$$

When $n = 6$, $m = \dfrac{372 - 180}{48} = 4$

4 big boxes and 6 small boxes can hold a total of 372 marbles.

20. In the total area of A, B and C, the areas of $a + d$, $b + d$ and $c + d$ have been counted twice. However, when they are removed from the total area of A, B and C, the area of d is removed as well. Therefore, the area of the whole figure
$= 40 + 50 + 60 - 12 - 14 - 16 + 8$
$= \mathbf{116\ cm^2}$

21. Method 1:



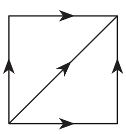
$1612 = 2 \times 2 \times 13 \times 31$
$= 2 \times 13 \times 2 \times 31$
$= 26 \times 62$
$26 + 62 = 88$
Therefore, the two numbers are **26** and **62**.

Method 2:
$1 + 87 = 88$, $\quad 1 \times 87 = 87$
$2 + 86 = 88$, $\quad 2 \times 86 = 172$
$\vdots \qquad\qquad \vdots$
$26 + 62 = 88$, $\quad 26 \times 62 = 1612$
The two numbers are **26** and **62**.

22. Let $a = \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ and $b = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$.

$$(1 + a)(b) - (1 + b)(a) = b + ab - a - ab$$
$$= b - a$$
$$= \mathbf{\frac{1}{5}}$$

23. There are 3 ways to move in each "box":



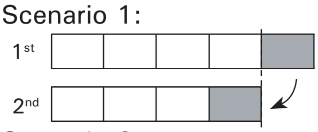
Therefore, there are $3 \times 3 \times 3 =$ **27 ways** to move from A to B.

24. If he was travelling on the scooter from the beginning, he would have travelled
$18 \times 4 = 72$ km.
Difference in distance travelled $= 72 - 60$
$= 12$ km
Difference in speed $= 18 - 6$
$= 12$ km/h
Time taken for walking $= 12 \div 12$
$= 1$ h
Distance walked $= 1 \times 6$
$=$ **6 km**

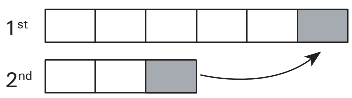
25. Method 1:
Let the number of oranges in the first and second baskets be a and b respectively.

$a - 1 = b + 1$ ----- (1)
$a + 1 = 3(b - 1)$ ----- (2)
From (2):
$a + 1 = 3b - 3$
$a - 3b + 4 = 0$ ----- (3)
From (1):
$a - b - 2 = 0$ ----- (4)
(3) – (4):
$a - 3b + 4 - a + b + 2 = 0$
$-2b + 6 = 0$
$2b = 6$
$b = 6 \div 2 = 3$
From (4):
$a - 3 - 2 = 0$
$a - 5 = 0$
$a = \mathbf{5}$

Method 2:
Scenario 1:

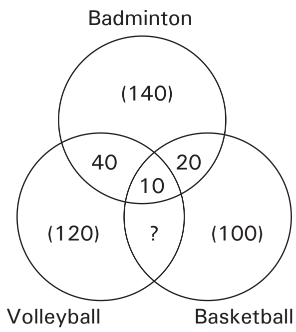


Scenario 2:

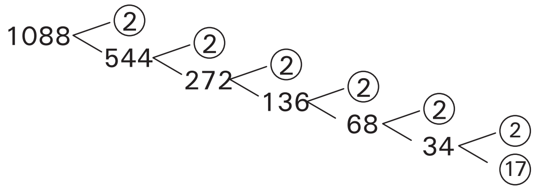


Therefore, there are **5** oranges in the first basket and **3** oranges in the second basket.

26.



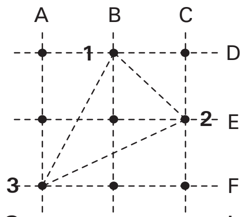
From the diagram, we have
$140 + 120 + 100 - (40 + 10) - (20 + 10) - (? + 10) + 10 = 250$, as the total number of students surveyed is 250.
But $(40 + 10)$ and $(20 + 10)$ have been counted twice, thus we deduct them. "10", on the other hand, has been deducted twice, thus we need to add it back.
$280 - ? = 250$
$? =$ **30** liked both volleyball and basketball, but not badminton.

27. 

$1088 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 17$
$1088 = 1 \times 1088 \qquad 1088 = 2 \times 544$
$1088 = 4 \times 272 \qquad 1088 = 8 \times 136$
$1088 = 16 \times 68 \qquad 1088 = 32 \times 34$
$1088 = 64 \times 17$
There are **7 ways** to form rectangle.

28. Factorise each term as follows:

$$\frac{(1 \times 2 \times 3)(1 + 2 + 3 + \ldots + 100)}{(1 \times 3 \times 5)(1 + 2 + 3 + \ldots + 100)}$$
$$= \frac{1 \times 2 \times 3}{1 \times 3 \times 5}$$
$$= \mathbf{\frac{2}{5}}$$

29. 

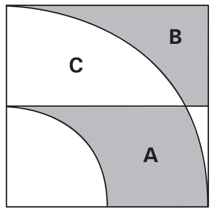
Suppose we use side A as the base of the triangle, there will be ${}^3C_2 = \frac{3 \times 2}{1 \times 2} = 3$ different bases.
We can choose a point from the six points on B and C as the other vertex. Therefore, there are ${}^6C_1 = 6$ different ways to choose the point.
$3 \times 6 = 18$ triangles
We can do the same for sides B and C,
$3 \times 18 = 54$ triangles
A triangle can also be formed using points 1, 2 and 3 as vertices as shown in the figure above. There are 4 ways to form such triangles.
Using lines D, E and F as base, $2 \times 3 \times 3 = 18$ ways.
Therefore, there are altogether $54 + 4 + 18$
$=$ **76 triangles.**

30. If the time taken to travel is t, the time taken to return is $(6 - t)$.
The distance travelled for each trip was the same.
Distance $= 1500t$
Returned distance $= 1200(6 - t) = 7200 - 1200t$
We can equate the two equations:
$1500t = 7200 - 1200t$
$2700t = 7200$
$$t = \frac{7200}{2700} = \frac{8}{3} = 2\frac{2}{3} \text{ h}$$
The plane travelled $1500 \times \frac{8}{3}$
$=$ **4000 km** before it made its return.

31. The teacher had to be born in 19ab, where a and b are two unknown 1-digit numbers.

$$2008 - 19ab = 1 + 9 + a + b$$
$$2008 - (1900 + 10a + b) = 10 + a + b$$
$$108 - 10a - b = 10 + a + b$$
$$98 = 11a + 2b$$
$$11a = 98 - 2b$$
$$a = \frac{98 - 2b}{11}$$

When $b = 5$, $a = \frac{98 - 10}{11} = \frac{88}{11} = 8$.
$2008 - 1985 = 23$ or $1 + 9 + 8 + 5 = 23$
The teacher was **23 years old** in 2008.

32. 

$$A + C = \frac{1}{4}(14)^2\pi - \frac{1}{4}(7)^2\pi$$
$$= \frac{1}{4}(196 - 49)\pi$$
$$= \frac{1}{4} \times 147 \times \frac{22}{7}$$
$$= 115.5 \text{ cm}^2$$

$B + C = 7 \times 14 = 98 \text{ cm}^2$
$(A + C) - (B + C) = A + C - B - C$
$= A - B$
$= 115.5 - 98$
$=$ **17.5 cm²**

33. $1^2 + 2^2 + 3^2 + \ldots + 15^2$
$= 15 \times (15 + 1) \times (2 \times 15 + 1) \div 6$
$= 15 \times 16 \times 31 \div 6$
$=$ **1240**

34. Let a be 12 345 678.

$$\frac{a}{a^2 - (a - 1)(a + 1)} = \frac{a}{a^2 - (a^2 - a + a - 1)}$$
$$= \frac{a}{a^2 - a^2 + 1}$$
$$= a$$
$$= \mathbf{12\ 345\ 678}$$

35. It suffices to trace the pattern of the remainders.

n	1	2	3	4	5	6	7	8	9	10	11	12
Remainder when $2^n \div 6$	2	4	2	4	2	4	2	4	2	4	2	4
Remainder when $n^2 \div 6$	1	4	3	4	1	0	1	4	3	4	1	0
Remainder when $2^n - n^2 \div 6$	1	0	5	0	1	4	1	0	5	0	1	4

The remainders recur at 1, 0, 5, 0, 1, 4, ...
$2019 \div 6 = 336$ R 3
$336 \times 2 + 1 =$ **673**

36. Let the distance for the whole journey be d metres.
Time taken by Alan
$$= \frac{d}{2} \div 4.5 + \frac{d}{2} \div 5.5$$
$$= \frac{d}{9} + \frac{d}{11}$$
$$= \frac{20d}{99} \text{ h}$$
Time taken by Benny
$= d \div [(4.5 + 5.5) \div 2]$
$$= \frac{2}{10}d \text{ h}$$
$= \frac{1}{5}d$ h where d is a constant.

Since $\frac{20}{99} > \frac{1}{5}$, **Benny** would arrive at the finishing line first since Alan took a longer time.

37. Let the number of marbles that Don and Andy have at first be a and b respectively.
Let c be the number of marbles given away.

$a - c = 2(b + c)$ ----- (1)
$a + c = 4(b - c)$ ----- (2)

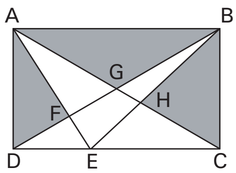
From (1):
$a - c = 2b + 2c$
$a - 2b - 3c = 0$ ----- (3)

From (2):
$a + c = 4b - 4c$
$a - 4b + 5c = 0$ ----- (4)

(4) – (3):
$-2b + 8c = 0$
$2b = 8c$
$b = 8, c = 2,$
$a = 22$

Don has **22** and Andy has **8 marbles** at first.

38.



Area of rectangle ABCD $= 20 \times 10$
$= 200$ cm^2

Area of $\triangle$ACE + Area of $\triangle$BDE
$= \frac{1}{2} \times$ Area of rectangle ABCD
$= 100$ cm^2

But the area of EFGH was counted twice.
$100 - 15 = 85$ cm^2 is non-shaded area
$200 - 85 = 115$ cm^2
Area of the shaded region is **115 cm²**.

39. Represent the 9 numbers as products of prime numbers, as follows:

$1, 2, 3, 2 \times 2, 5, 2 \times 3, 7, 2 \times 2 \times 2, 3 \times 3$

Colin $= 1 \times 9 \times 7 = 63$
Bernard $= 4 + 5 + 6 = 15$
Alice $= 2 \times 3 \times 8 = 48$

Therefore,
Alice → **2, 3, 8**
Bernard → **4, 5, 6**
Colin → **1, 9, 7**

40. We can write each term as follows:

$$31\frac{1}{2} \times \frac{2}{3} = \left(30 + \frac{3}{2}\right) \times \frac{2}{3} = 20 + 1$$

$$41\frac{1}{3} \times \frac{3}{4} = \left(40 + \frac{4}{3}\right) \times \frac{3}{4} = 30 + 1$$

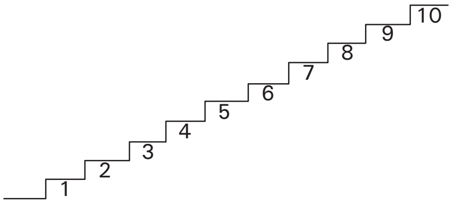
$$51\frac{1}{4} \times \frac{4}{5} = \left(50 + \frac{5}{4}\right) \times \frac{4}{5} = 40 + 1$$

$$61\frac{1}{5} \times \frac{5}{6} = \left(60 + \frac{6}{5}\right) \times \frac{5}{6} = 50 + 1$$

$$31\frac{1}{2} \times \frac{2}{3} + 41\frac{1}{3} \times \frac{3}{4} + 51\frac{1}{4} \times \frac{4}{5} + 61\frac{1}{5} \times \frac{5}{6}$$

$= 20 + 1 + 30 + 1 + 40 + 1 + 50 + 1$
$=$ **144**

41.



The following are the possible ways to go from the first step to the 10th step:

$3 \to 3 \to 2 \to 2$
$3 \to 2 \to 3 \to 2$
$3 \to 2 \to 2 \to 3$
$2 \to 3 \to 2 \to 3$
$2 \to 2 \to 3 \to 3$
$2 \to 3 \to 3 \to 2$
$2 \to 2 \to 2 \to 2 \to 2$

There are altogether **7 ways**.

42. Scenario 1:
When speed is increased by 20%,
Ratio of speed $= 120 : 100 = 6 : 5$
Hence, ratio of time $= 5 : 6$.
$6 - 5 = 1$
$1 \text{ u} = 1 \text{ h}$
$6 \text{ u} = 6 \times 1 = 6 \text{ h}$
The time taken is 6 hours if the car travels at its original speed.

Scenario 2:
When speed is increased by 25%,
Ratio of speed $= 100 : 125 = 4 : 5$
Hence, ratio of time $= 5 : 4$
Time saved $= \frac{36}{60} = \frac{3}{5}$ h
$5 - 4 = 1$
1 unit $= \frac{3}{5}$ h
5 units $= 5 \times \frac{3}{5} = 3$ h
Time taken for the first 120 km is 3 h.
Speed for the first 120 km
$120 \div 3 = 40$ km/h
Distance between the two towns $= 40 \times 6$
$=$ **240 km**

43. Let b, m and s be the number of big rooms, mid-sized rooms and small rooms respectively.
Hence, we have

$b + m + s = 12$ ----- (1)
$8b + 7m + 5s = 80$ ----- (2)

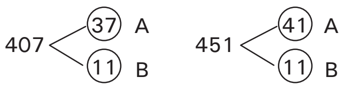
(1) × 5:
$5b + 5m + 5s = 60$ ----- (3)

(2) – (3):
$3b + 2m = 20$
$3b = 20 - 2m$
$b = \frac{20 - 2m}{3}$

When $m = 4$, $b = \frac{20 - 8}{3} = 4$
From (1): $s = 12 - 4 - 4 = 4$
There are **4 big rooms**, **4 mid-sized rooms** and **4 small rooms.**
Another possible answer: $m = 1, b = 6, s = 5$
There are **6 big rooms**, **1 mid-sized room** and **5 small rooms.**
Also, $b = 2, m = 7, s = 3$

44. We find the multiples of 3 or 5 first. The common multiples of 3 and 5 (i.e. 15, 30, ...) have been counted twice.
Number of multiples of 3 $= 200 \div 3 = 66$ R 2
Number of multiples of 5 $= 200 \div 5 = 40$
Multiples of 3 and 5 are multiples of $3 \times 5 = 15$.
Number of multiples of 15 $= 200 \div 15$
$= 13$ R 5
Number of multiples of 3 or 5 $= 66 + 40 - 13$
$= 93$
$200 - 93 = 107$ numbers are not multiples of 3, 5 or 15.
We list these numbers from the largest to the smallest instead.
199, 197, 196, 194, 193, 191,
188, 187, 184, 182, 181, 179,
178, ...
Since $107 - 95 = 12$, the 95th number is **178** by counting backwards.

45.



$407 = 37 \times 11$ $\quad 451 = 41 \times 11$
David had misread "1" as "7" and Sophia had misread "3" as "4".
Therefore, A = 31.
$A \times B = 31 \times 11 =$ **341**

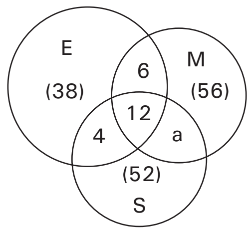
46. There are 12 odd numbers, 11 even numbers.
Scenario 1: Even and Even
$$^{11}C_2 = \frac{11 \times 10}{1 \times 2} = 55$$
Scenario 2: Odd and Odd
$$^{12}C_2 = \frac{12 \times 11}{1 \times 2} = 66$$
$55 + 66 =$ **121 ways**

47. $4^{11} = 2^{2 \times 11}$
$= 2^{22}$
$2 \times 5 = 10$
$$2^{22} \times 5^{20} = 2^2 \cdot 2^{20} \cdot 5^{20} = 4\underbrace{00 \cdots 00}_{20 \text{ “0”}}$$
Ans: 21

48. Let his driving speed and the speed of the train be m and n respectively.
Distance $= 15m + 20n$
Distance $= 8m + 34n$
We can equate the two equations above as follows:
$15m + 20n = 8m + 34n$
$7m = 14n$
$m = 2n$
His driving speed is twice the speed of the train.
Minimum time spent on driving $= 15 \times 1 + 20 \times \frac{1}{2}$
$= 15 + 10$
$= 25$ h
He must drive for at least **25 hours**.

49. Let f, n and b be the number of fiction books, non-fiction books and bibliographies respectively.
$f + n + b = 30$ ----- (1)
$\frac{f}{3} + \frac{n}{2} + 3b = 30$ ----- (2)
(2) $\times$ 6:
$2f + 3n + 18b = 180$ ----- (3)
(1) $\times$ 2:
$2f + 2n + 2b = 60$ ----- (4)
(3) – (4):
$n + 16b = 120$
$n = 120 - 16b$
When $b = 7$, $n = 120 - 112 = 8$
From (1): $f + 8 + 7 = 30$
$f + 15 = 30$
$f = 15$
She read **15 fiction books** and **8 non-fiction books**.

50. (a)

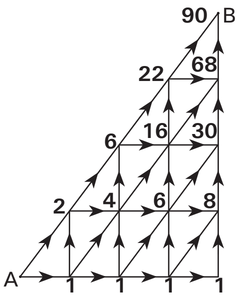


$100 = 56 + 38 + 52 - 6 - 12 - 4 - 12 - a - 12 + 12$
$= 112 - a$
$a = 12$
12 students liked both Mathematics and Science.
(b) $56 - 6 - 12 - 12 = 26$
26 students liked only Mathematics.

51. Let the first 3 digits of the number be abc. Hence, his mobile phone number is abcabc0.
abcabc0 = abcabc $\times$ 10
= abc $\times$ 1001 $\times$ 10
= abc $\times 7 \times 11 \times 13 \times 2 \times 5$
abc $= 3 \times 17 \times 19 = 969$
Since the number is a product of a series of consecutive prime numbers (that is, 2, 3, 5, 7, 11, 13, 17 and 19), his mobile phone number is **9699690**.

52. $\frac{1}{2 \times 4} = \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{4}\right)$
$\frac{1}{4 \times 6} = \frac{1}{2} \times \left(\frac{1}{4} - \frac{1}{6}\right)$
$\vdots \qquad \vdots$
$\frac{1}{98 \times 100} = \frac{1}{2} \times \left(\frac{1}{98} - \frac{1}{100}\right)$
$$A = \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + \ldots + \frac{1}{98} - \frac{1}{100}\right)$$
$$= \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{100}\right)$$
$$= \frac{1}{2} \times \frac{49}{100}$$
$$= \mathbf{\frac{49}{200}}$$

53.



There are **90 ways** to reach B from A.

54. Assume that the whole distance is "1".

In the 1950s, speed $= \dfrac{1}{15.6}$

In the 1960s, speed $= \dfrac{1}{15.6} \times 1.30$

In the 1970s, speed $= \dfrac{1}{15.6} \times 1.30 \times 1.20$

$= \dfrac{1.56}{15.6}$

$= \dfrac{1}{10}$

The train took **10 h** in the 1970s.

55. Method 1:

$$\begin{array}{r} a\ b\ c\ d\ e \\ +\quad f\ g\ h \\ \hline 6\ 8\ 2\ 4\ 7 \\ \hline \end{array} \qquad \begin{array}{r} a\ b\ c \\ +d\ e\ f\ g\ h \\ \hline 3\ 6\ 0\ 9\ 0 \\ \hline \end{array}$$

a = 6 and d = 3 as there are no carry-over.
Since 5 + 2 = 7, e = 5 and h = 2.
Since 8 + 2 = 10 and 1 + 7 + 1 = 9, c = 8, b = 7 and g = 1.
Since 8 + 4 = 12, f = 4.
Uncle Richard's phone number is **67835412**.

Method 2:

Let the first 3 digits be x, the last 3 digits be y and the two numbers in the middle be n.

$100x + n + y = 68247$ ----- (1)
$x + 1000n + y = 36090$ ----- (2)
(1) – (2):
$99x - 999n = 32157$
$99x = 32157 + 999n$
$x = \dfrac{32157 + 999n}{99}$

When $n = 35$, $x = \dfrac{67122}{99} = 678$
From (2),
$678 + 35000 + y = 36090$
$y = 36090 - 678 - 35000$
$y = 412$
Uncle Richard's phone number is **67835412**.

56. Number of multiples of 2 = 2019 ÷ 2
= 1009 R 1
Number of multiples of 3 = 1009 ÷ 3
= 336 R 1
Number of multiples of 7 = 1009 ÷ 7
= 144 R 1
Multiples of 21 = 1009 ÷ 21
= 48 R 1
Number of multiples of 2 that are not divisible by 3 or 7 = 1009 – 336 – 144 + 48
= **577**

57. Establish a pattern for the remainders when each term is divided by 3.

Term	1	1	2	3	5	8	13	21
Remainder	1	1	2	0	2	2	1	0

Term	34	55	89	144	233	377	610	987
Remainder	1	1	2	0	2	2	1	0

The remainders repeat in the pattern 1, 1, 2, 0, 2, 2, 1 and 0 (that is, after every 8 terms).

1	1	2	0	2	2	1	0
↓	↓	↓	↓	↓	↓	↓	↓
R 1	R 2	R 3	R 4	R 5	R 6	R 7	R 0

2008 ÷ 8 = 251 R 0
Therefore, the remainder is **0** when the 2008th term is divided by 3.

58. $\dfrac{(1 + 199) \times 100 \div 2}{(2 + 200) \times 100 \div 2} = \dfrac{200}{202} = \mathbf{\dfrac{100}{101}}$

59. It is sufficient to consider the first 3 numbers as 5-digit palindromes are in the form of abcba.

10 thousands	thousands	hundreds
☐	☐	☐
↓	↓	↓
2, 4, 6, 8	**1 to 10**	**1 to 10**

This means that the digits in the thousands and hundreds places can be repeated.
4 × 10 × 10 = 400
There are **400** such palindromes.

60. Ratio of speed = 36 : 24 = 3 : 2
We assume that the running speed and the walking speed are 3 steps and 2 steps respectively.
Let the number of steps be n.

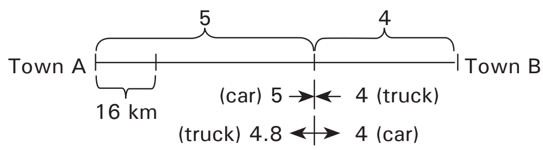
$\dfrac{n-24}{3} + \dfrac{24}{2} = 28$

$\dfrac{n-24}{3} = 16$

$n - 24 = 48$
$n = 48 + 24$
$= 72$

There were **72 steps** on the staircases.

61. Speed of car after it met the truck
= 5 × (100% – 20%)
= 4
Speed of truck after it met the car
= 4 × (100% + 20%)
= 4.8



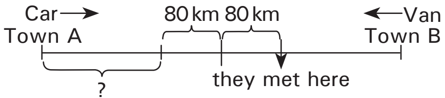
5 – 4.8 = 0.2 units
16 ÷ 0.2 = 80 km
1 unit = 80 km
5 + 4 = 9 units
9 units = 80 × 9 = 720 km
Town A was **720 km** from Town B.

72. A table can help to trace the pattern of distribution.

Region	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th
R 1 → R 5	✓		✓			✓		✓			✓
R 6 → R 9		✓			✓		✓			✓	
R 10 → R 14	✓			✓		✓			✓		✓
R 15 → R 18			✓		✓			✓		✓	
R 19 → R 22		✓		✓			✓		✓		
	✓		✓			✓		✓			✓

The pattern repeats after every 22 parcels.
$2009 \div 22 = 91$ R 7
Therefore, the **5th region** will get the last parcel.

73. The car travelled $80 \times 2 = 160$ km more than the van when they met.



$1.4 - 1 = 0.4$
0.4 times → 160 km
1 time → $160 \div 0.4 = 400$ km
Hence, distance travelled by the van = 400 km
Distance between Town A and Town B
$(400 + 80) \times 2 =$ **960 km**

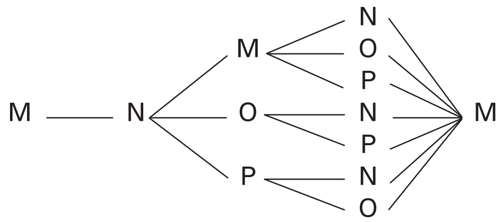
74. Let the number of students in the classes be A, B, C and D respectively.
$B + C + D = 122$ ----- (1)
$A + B + C = 123$ ----- (2)
$2 + B + C = A + D$ ----- (3)
(1) + (2):
$2B + 2C + A + D = 245$ ----- (4)
Substitute (3) into (4):
$2B + 2C + 2 + B + C = 245$
$3B + 3C + 2 = 245$
$3(B + C) = 243$
$B + C = 81$
Substitute $B + C = 81$ into (3):
$A + D = 2 + 81$
$= 83$
$A + B + C + D = 81 + 83$
$= 164$
There are **164 students.**

75. Find the least common multiple (LCM) of 3, 4, 6 and 7.

$$\begin{array}{r|l} 3 & 3,\ 4,\ 6,\ 7 \\ \hline 2 & 1,\ 4,\ 2,\ 7 \\ \hline & 1,\ 2,\ 1,\ 7 \end{array}$$

LCM $= 3 \times 2 \times 2 \times 7$
$= 84$
If the farmer has $84 - 1 = 83$ eggs,
$83 \div 3 = 27$ R 2
$83 \div 4 = 20$ R 3
$83 \div 6 = 13$ R 5
$83 \div 7 = 11$ R 6
Therefore, the farmer has at least **83 eggs**.

76.



From the above figure, Melvin passes the ball to Nigel and Nigel passes the ball to one of the three of the rest, and so on. Since Melvin can also start passing the ball to Owen or Patrick, we can draw two such figures to illustrate these two different situations. Hence, we have
$7 \times 3 = 21$.
There are **21 ways** to pass the ball.

77. (a)

$$\begin{array}{r} 8\ 4 \\ \times\ 4\ 8 \\ \hline 6\ 7\ 2 \\ 3\ 3\ 6\ \ \\ \hline 4\ 0\ 3\ 2 \end{array}$$

A = 8, B = 4

(b)

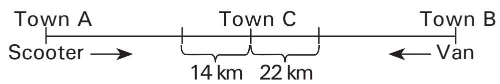
$$\begin{array}{r} 5\ 4\ 7\ 2 \\ -\ 2\ 5\ 4\ 7 \\ \hline 2\ 9\ 2\ 5 \\ \hline \end{array}$$

A = 5, B = 4, C = 7, D = 2

78. (a) The last number of the 9th row is $1 + 2 + 3 + \ldots + 9 = 45$.
Therefore, the 3rd number from the left in the 10th row is 48.

(b) Last number in the 62nd row is $1 + 2 + 3 + \ldots + 62$
$= (62 + 1) \times 62 \div 2$
$= 1953$
The 63rd row starts with 1954, 1955, 1956, ..., 2016.
$2016 - 2009 + 1 = 8$
2009 is the 8th from the right of the 63rd row.
Hence, it is the $63 - 8 + 1 =$ **56th number from the left of the 63rd row.**

79. In scenario 2 and scenario 3, the combined speed is the same.



$14 + 22 = 36$
Difference in speed = 8 km/h
The scooter takes $36 \div 8 = 4.5$ h to travel from Town A to 14 km from Town C.
Hence, the scooter travels 14 km in $5 - 4.5 = 0.5$ h.
Speed of scooter $14 \div 0.5 =$ **28 km/h**

80. The give-away of this question is the price of each table.
Price of each table $= 320 \div 5 = \$64$
$c \times 5 - 3 \times 64 = 48$
where c is the price of each chair.
$c = \$48$
$320 \div (64 - 48) = 20$ chairs.
Mr Shapiro has **20 chairs.**

81. **(a)** Using prime factorisation, we factorise 75 and 120 as follows:

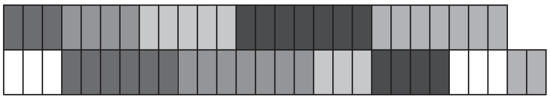
$$\begin{array}{r|ll} 5 & 75, & 120 \\ \hline 3 & 15, & 24 \\ \hline & 5, & 8 \end{array}$$

$5 \times 3 = 15$
The length of each side is **15 m**.

(b) There are $5 \times 8 =$ **40 squares**.

82. The strategy is to shorten the waiting time of Team B. Hence, we start with Task 3 so that Team B only needs to wait for 3 h (the least number of hours). This is followed by Task 5, Task 1, Task 2, and finally Task 4, as follows:
Beginning with Team A.

Task no.	3	→	5	→	1	→	2	→	4
	↓		↓		↓		↓		↓
Team A	3 h		4 h		5 h		7 h		7 h
	↓		↓		↓		↓		↓
Team B	6 h		7 h		3 h		4 h		2 h



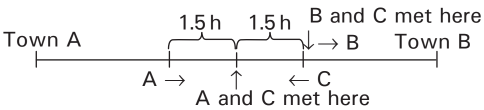
83. Let the 5 other digits of the original number be a, b, c, d and e respectively. Hence, the original number is 1 a b c d e.
Hence, we have the following.

$$\begin{array}{r} 1\ a\ b\ c\ d\ e \\ \times \qquad\quad 3 \\ \hline a\ b\ c\ d\ e\ 1 \\ \hline \end{array}$$

e = 7 since $7 \times 3 = 21$
d = 5 since $3 \times 5 = 15$ and $15 + 2 = 17$
c = 8 since $3 \times 8 = 24$ and $24 + 1 = 25$
b = 2 since $2 \times 3 = 6$ and $6 + 2 = 8$
a = 4 since $4 \times 3 = 12$
Therefore, my social Identity number is **142857**.

84. We can see that only a fraction has denominator 1, 2 fractions have denominator 2, 3 fractions have denominator 3, and so on. Number of fractions with denominators 29 to $1 + 2 + 3 + ... + 29$
$= (1 + 29) \times 29 \div 2$
$= 15 \times 29$
$= 435$
$435 + 16 = 451$
Therefore, there are **450 fractions** before $\frac{16}{30}$.

85. It is sufficient to consider the distance between the two meeting points.



$1.5 \times (80 + 60) = 210$ km

210 km is also the distance between Car A and Car B.
$70 - 60 = 10$
Car A was behind Car B, for 10 km every hour.
$210 \div 10 = 21$
21 hours was the time taken for Car B and Car C to meet.
Distance between Town A and Town B
$21 \times (80 + 70) =$ **3150 km**

86. USD1 → S\$120 ÷ 80 = S\$1.50
USD1 → 15 000 VD
Hence, S\$1.50 → 15 000 VD
S\$1 → 15 000 ÷ 1.5 = 10 000 VD
Since S\$1 can be exchanged for 10 000 VD, 20 000 VD can be exchanged for **\$2**.

87. Note that if $a \div m$ and $b \div m$ leave the same remainder, then $a - b$ is divisible by m.
$1596 - 1238 = 2 \times 179$
$2491 - 1596 = 895$
$\qquad\qquad\quad = 5 \times 179$
Now, $1238 = 6 \times 179 + 164$
Thus $n = 164$
$m + n = 179 + 164$
$\qquad\quad =$ **343**

88.

	Glass 1	Glass 2
1st	$140 - 140 \times \frac{1}{2} = 70$ ml	$140 + 70 = 210$ ml
2nd	$70 + 70 = 140$ ml	$210 - \frac{1}{3} \times 210 = 140$ ml
3rd	$140 - 140 \times \frac{1}{4} = 105$ ml	$140 + 35 = 175$ ml
4th	$105 + 35 = 140$ ml	$175 - \frac{1}{5} \times 175 = 140$ ml

The pattern repeats after every other time. The two glasses contain 140 ml of milk each after every even number of times of pouring.
Therefore, there is **140 ml** of milk in each glass after pouring 100 times.

89.

$$\begin{array}{r} a\ b\ c\ d\ e\ 7 \\ \times \qquad\quad 5 \\ \hline 7\ a\ b\ c\ d\ e \\ \hline \end{array}$$

By working out the product above, we get e = 5, d = 8, c = 2, b = 4 and a = 1.
The 6-digit number is **142857**.

90. $\underbrace{111 \cdots 1}_{207\ 1\text{'s}} \times \underbrace{111 \cdots 1}_{207\ 1\text{'s}}$

$= \underbrace{111 \cdots 1}_{207\ 1\text{'s}} \times (1\underbrace{000 \cdots 0}_{207\ 0\text{'s}} - 1) \div 9$

$= (\underbrace{111 \cdots 1}_{207\ 1\text{'s}}\ \underbrace{000 \cdots 0}_{207\ 0\text{'s}} - \underbrace{111 \cdots 1}_{207\ 1\text{'s}}) \div 9$

$= \underbrace{111 \cdots 1}_{206\ 1\text{'s}}\ 0\ \underbrace{888 \cdots 8}_{206\ 8\text{'s}}9 \div 9$

$= \underbrace{123456790 \cdots 123456790}_{22 \text{ of } 123456790}\ 12345678$
$\quad \underbrace{987654320 \cdots 987654320}_{22 \text{ of } 987654320}\ 987654321$

$81 \times 23 =$ **1863**

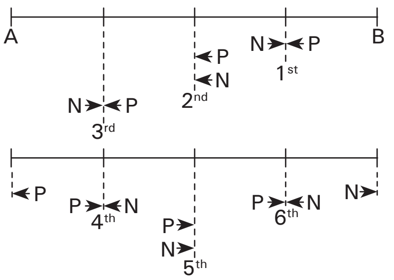
123. Let Harry's age be h. Hence, Paul's age is $\frac{5}{4}$ h.

$$8(h + 9) = 7\left(\frac{5}{4}h + 9\right)$$
$$8h + 72 = \frac{35}{4}h + 63$$
$$72 - 63 = \frac{35}{4}h - \frac{32}{4}h$$
$$\frac{3}{4}h = 9$$
$$h = 9 \times \frac{4}{3} = 12$$

Harry is **12 years** old now.

124. Let the number be $\overline{a123}$.
Hence, (123 – a) is divisible by 13.
When a = 6, 123 – 6 = 117
117 ÷ 13 = 9
Hence, the smallest value of the product is 6123.
Smallest possible value of n = 6123 ÷ 13 = **471**

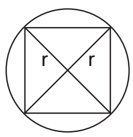
125.



Piere will cover **$1\frac{3}{4}$ single trips** in all.

126. Each question was answered correctly 3 times by participants A to E.
26 + 21 + 32 + 26 + 21 = 126
126 ÷ 3 = 42
F scored **42** points.

127.



Each △ has an area of $\frac{50}{4}$ = 12.5 cm^2.

Area of △ = $\frac{1}{2}$(r)(r) = 12.5
r^2 = 25
r = 5

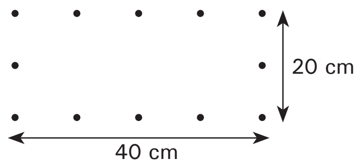
Area of the circle = πr^2
= 3.14 × 5 × 5
= **78.5 cm^2**

128. Let the house number be $\overline{abcd}$. Hence, the palindrome is $\overline{dcba}$.
$\overline{dcba} - \overline{abcd} = 7263$
1000d + 100c + 10b + a – 1000a – 100b – 10c – d = 7263
999d + 90c – 90b – 999a = 7263
9(111d + 10c – 10b – 111a) = 7263
111d + 10c – 10b – 111a = 807
When d = 8 and a = 1,
888 + 10c – 10b – 111 = 807
10c – 10b = 807 – 777 = 30
c – b = 3
Hence, c = 6 and b = 3.
His house number is **1368**.

129. Since 2 + 9 + 16 + 23 + 30 = 80, the Saturdays fell on 2 March, 9 March, 16 March, 23 March and 30 March.
9 March was also a **Saturday**.

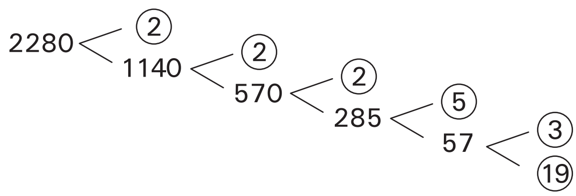
130. We should capitalise on the strength of each factory.
Factory A:
480 ÷ 20 = 24 pairs of gloves/day
480 ÷ 10 = 48 pairs of boots/day
The strength of factory A is to produce 48 pairs of boots a day.
Factory B:
560 ÷ 14 = 40 pairs of gloves/day
560 ÷ 16 = 35 pairs of boots/day
The strength of factory B is to produce 40 pairs of gloves a day.
Factory A:
48 × 27 = 1296 pairs of boots in 27 days
24 × 3 = 72 pairs of gloves in 3 days
Factory B:
30 × 40 = 1200 pairs of gloves
72 + 1200 = 1272
The factories can jointly produce a maximum **1272 sets**.

131.



Using the width as base, there are 5.
5 × 2 = 10 altogether
Using the length as base, there are 10, but 4 repeat.
10 – 4 = 6
6 × 2 = 12 altogether
There are **22 triangles**.

132. **(a)**

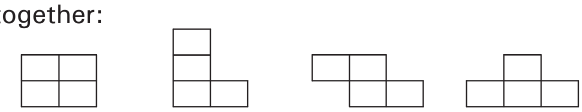


$$2280 = 2 \times \underbrace{2 \times 2 \times 3}_{12} \times \underbrace{5 \times 19}_{95}$$

Therefore, Lucas was **12 years old**.

(b) He came in second in the competition with a score of 95.

133. Number of ways the 4 stamps can be joined together:



3 of this | 9 of this | 6 of this | 6 of this

There are altogether **24 ways** to do so.

360° is equivalent to 1 round. The ball is rolled 1 round along each side.
$3 \times 1 + 1 = 4$
The ball will roll a total of **4 rounds**.

152. (a) Treat the whole project as 1.

Combined rate $= \frac{1}{20}$

Portion of work left $= 1 - \frac{1}{20} \times 8$
$= 1 - \frac{2}{5}$
$= \frac{3}{5}$

Since Team B finishes the remaining project in 18 days, $18 \div \frac{3}{5} = 18 \times \frac{5}{3} = 30$
Team B can finish the whole project in **30 days**.

(b) Rate of Team B $= \frac{1}{30}$

Rate of Team A $= \frac{1}{20} - \frac{1}{30} = \frac{1}{60}$

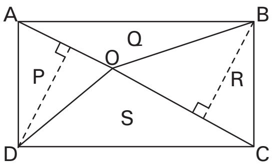
It takes Team A $1 \div \frac{1}{60}$ = **60 days** to complete the whole project.

153. The angle formed between the hour hand and the minute hand at 4 o'clock is 360° ÷ 3 = 120°. The first 30° is formed when the time is past 4.15. We want to find the time taken by the minute hand to catch up to 120° – 30° = 90°.
Suppose the minute hand takes n minutes to catch up.
The minute hand travels 360° ÷ 60 × n = 6° × n and the hour hand travels 360° ÷ (12 × 60) × n = 0.5° × n in a minute.
Angle to catch up = 90°
$6n - 0.5n = 90$
$5.5n = 90$
$n = \frac{90}{5.5}$
$= \frac{180}{11}$
$= 16\frac{4}{11}$

They first form 30° after **$16\frac{4}{11}$ minutes**.

154. From the information given, either B or C is the youngest.
A + B + C + D = 62 ----- (1)
B + C + 8 = A + D ----- (2)
Substitute (2) into (1):
B + C + 8 + B + C = 62
2B + 2C = 62 – 8 = 54
B + C = 54 ÷ 2 = 27
A + D = 62 – 27 = 35
Hence, either A or D is the eldest.
Since 35 = 17 + 18, his age is **18**.

155.

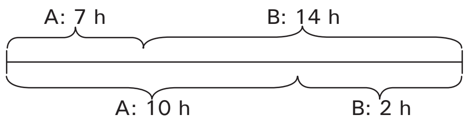


Area of Region P = area of Region Q as OA is the common base and they have the same height.
Area of Region P + area of Region Q
= 30 + 30
= 60 cm²
Since area of Region S = area of Region R,
60 cm² → 100% – 35% – 35% = 30%
30% → 60 cm²
1% → 60 ÷ 30 = 2 cm²
100% → 2 × 100 = 200 cm²
Area of rectangle = **200 cm²**

156. Ratio of speed = 40 : 60
= 2 : 3
Ratio of time = 3 : 2
Time taken to travel from Town A to Town B
$= \frac{3}{5} \times 15$
= 9 h
Time taken to return to Town A from Town B
= 15 – 9
= 6 h
Distance between Town A and Town B
= 40 × 9
= 360 km or 60 × 6
= **360 km**

157. Length of sides of the squares from the smallest to the largest: 3 cm, 6 cm, 9 cm, 12 cm, 21 cm, ...
Length of side of the shaded square
= 6 + 9 + 21
= 36 cm
Area of the shaded region
= 36 × 36
= **1296 cm²**

158. Use the method of substitution and comparison to solve this problem.



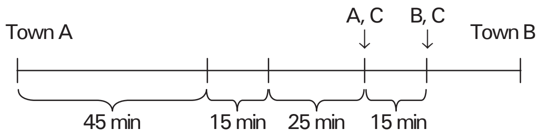
10 – 7 → 14 – 2
3 h → 12 h
1 h → 4 h
When Pipe A is turned on for 1 h, Pipe B works for 4 h.
7 × 4 + 14 = 42 h or 10 × 4 + 2 = 42 h are required by Pipe B to fill up the pool on its own.
42 – 5 × 4 = **22 hours** are needed by Pipe B when Pipe A is turned on for 5 h.

159. Suppose that it takes n minutes to form 90°.
The minute hand travels 360° ÷ 60 × n = 6° × n and the hour hand travels 360° ÷ 320 × n = 0.5° × n in a minute.
The minute hand and the hour hand form 90° when the time is past 4.35. Distance for the minute hand to catch up = 7 × 30 = 210°.

190. Total age of A, B, and C $= 19 \times 3 = 57$
Total age of B, C, and D $= 22 \times 3 = 66$
Total age of A, C and D $= 21 \times 3 = 63$
$A + B + C + B + C + D - (A + C + D)$
$= 57 + 66 - 63$
$= 60$
Hence, $2B + C = 60$
$B = \dfrac{60 - C}{2}$
When $C = 22$, $B = \dfrac{60 - 22}{2} = 19$
$A = 57 - 22 - 19 = 16$
$D = 63 - 16 - 22 = 25$
A is **16 years old**, B is **19 years old**, C is **22 years old** and D is **25 years old**.

191. Use the concept of excess and shortage to solve this problem.
Selling price $= (\$450 + \$350) \div (0.9 - 0.8)$
$= \$800 \div 0.1$
$= \$8000$
Cost price $= \$8000 \times 90\% - \$450 =$ **\$6750**
or
Cost price $\$8000 \times 80\% + 350 =$ **\$6750**

192. First, look for the ratio of speeds. Recall that the ratio of speed is the reverse of the ratio of time.



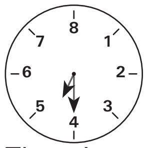
Car A : Car C $= 25 : 45 + 15 + 25$
$= 25 : 85$
$= 5 : 17$
Car B : Car C $= 25 + 15 : 15 + 45$
$= 40 : 60$
$= 2 : 3$
Hence, Car A : Car B : Car C $= 15 : 34 : 51$
Car B needs to cover the 45 minutes head start by Car A.
Time to catch up $= \dfrac{45 \times 15}{34 - 15}$
$= \dfrac{675}{19}$
$= 35\frac{10}{19}$ min
Therefore, Car B caught up with Car A **$30\frac{10}{19}$ minutes** after the start of its own journey.

193. $2\pi r = 20.4$
$r = \dfrac{20.4}{2\pi}$
$= \dfrac{10.2}{\pi}$ cm
Since area of the circle equals the area of rectangle,
$r \times r \times \pi = r \times L$
where L is the length of the rectangle
$L = r \times \pi$
$= \dfrac{10.2}{\pi} \times \pi$
$= 10.2$ cm

Perimeter of the shaded region
$= 10.2 \times 2 - r + r + \frac{1}{4}(2\pi r)$
$= 20.4 + \frac{1}{2}\pi \times \dfrac{10.2}{\pi}$
$= 20.4 + 5.1$
$=$ **25.5 cm**

194. Number of units of passengers who enter the gate in 9 minutes $= 1 \times 3 \times 9 = 27$ units
(Every unit contains a certain number of passengers.)
Number of units of passengers who enter the gate in 5 minutes $= 1 \times 5 \times 5 = 25$ units
Rate of queue $= (27 - 25) \div (9 - 5)$
$= 2 \div 4$
$= 0.5$ units/min
Number of waiting units until 8.05 am
$= 25 - (0.5 \times 5)$
$= 22.5$ units
or
$27 - (0.5 \times 9) = 22.5$ units
$22.5 \div 0.5 = 45$
The first person at the queue came 45 min before 8.05 am.
Therefore, the first passenger arrived at **7.20 am.**

195. The minute hand travels and shows 48 minutes while the hour hand travels $48 \div 8 = 6$ minutes in an hour.
The clock below shows the time 4.24.



The minute hand travels 180°.
The hour hand travels $\dfrac{360° \div 8}{2} = \dfrac{45°}{2} = 22.5°$.
The angle formed is **22.5°**.

196. $\dfrac{2018 + 2017 \times 2019}{2018 \times 2019 - 1} + \dfrac{2019 + 2018 \times 2020}{2019 \times 2020 - 1}$
$= \dfrac{2018 + 2017 \times 2019}{2017 \times 2019 + 2019 - 1} + \dfrac{2019 + 2018 \times 2020}{2018 \times 2020 + 2020 - 1}$
$= \dfrac{2018 + 2017 \times 2019}{2018 + 2017 \times 2019} + \dfrac{2019 + 2018 \times 2020}{2019 + 2018 \times 2020}$
$= 1 + 1$
$=$ **2**

197. Consider the worst case scenario where a marble of different colour is drawn each time.

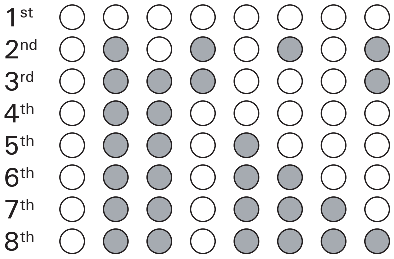
Blue	Yellow	Orange	White
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

$4 \times 4 = 16$
$16 + 1 = 17$
The 17^{th} marble drawn will give 5 marbles of the same colour. Therefore, **17 marbles** must be drawn.

198.
$1 \times 1 = 1$, $\quad 1 - 1 = 0$
$2 \times 2 = 4$, $\quad 4 - 1 = 3$
$3 \times 3 = 9$, $\quad 9 - 1 = 8$
$4 \times 4 = 16$, $\quad 16 - 1 = 15$
$5 \times 5 = 25$, $\quad 25 - 1 = 24$
$\vdots \quad\quad \vdots$

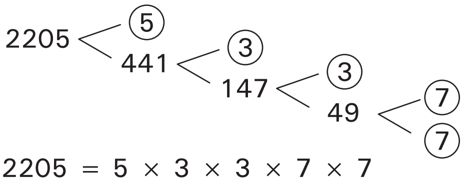
$10 \times 10 = 100, \quad 100 - 1 = 99$
Therefore, the 10th term is **99**.

199.



Therefore, there are **6 black beads** with the black surfaces facing in front.

200.



$2205 = 5 \times 3 \times 3 \times 7 \times 7$
$= 5 \times 3 \times 7 \times 3 \times 7$
Hence, the smallest value of n = **5**.

201. The product is maximum when it contains as many multipliers (which are greater than 1) as possible. Hence, we want to get as many "3" as possible.
$17 = 3 + 3 + 3 + 3 + 3 + 2$
$3 \times 3 \times 3 \times 3 \times 3 \times 2 = 486$
The product is maximum when it is **486**.

202. Let $x = 123457$

$= \dfrac{123321}{x^2 - (x - 1)(x + 1)}$

$= \dfrac{123321}{x^2 - x^2 + 1}$

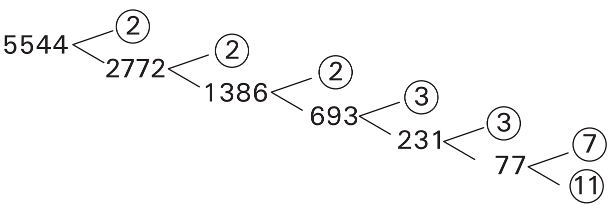
$= \mathbf{123321}$

203. There are $45 - 33 + 1 = 13$ different number of fruits in each box.
$106 \div 13 = 8 \text{ R } 2$
$8 + 1 = 9$
At least **9 boxes** have the same number of oranges.

204. $a + a + 3 = 2a + 3$
$2a + 3 = 13$
$2a = 13 - 3$
$= 10$
$a = 10 \div 2$
$= \mathbf{5}$

205. If A told the truth, then B, C and D lied.
If B told the truth, then A, C and D lied.
There is a contradiction, hence both A and B lied.
Since both A and B lied, C told the truth. It follows that D also lied.
Therefore, only **C** told the truth.

206.



$5544 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 11$
$= 8 \times 9 \times 7 \times 11$
Sum of ages $= 8 + 9 + 7 + 11$
$=$ **35 years**

207. We assume that the other four students score 139 each.
Total score of 4 students $= 139 \times 4$
$= 556$
Total score of 5 students $= 140 \times 5$
$= 700$
$700 - 556 = 144$
The highest possible score is **144**.

208. Let $A = \left(\dfrac{1}{11} + \dfrac{1}{21} + \dfrac{1}{31}\right)$

$B = \left(\dfrac{1}{11} + \dfrac{1}{21} + \dfrac{1}{31} + \dfrac{1}{41}\right)$

We want to see if the difference of M and N is greater than 0.
$(1 + A) \times (B) - (1 + B) \times (A)$
$= B + AB - A - AB$
$= B - A$

$= \dfrac{1}{11} + \dfrac{1}{21} + \dfrac{1}{31} + \dfrac{1}{41} - \dfrac{1}{11} - \dfrac{1}{21} - \dfrac{1}{31}$

$= \dfrac{1}{41} > 0$

Therefore, M > N. **M** has a greater value than N.

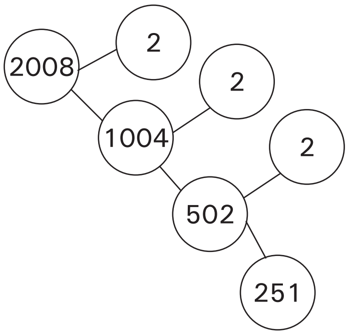
209. $134 - 124 + 1 = 11$
There are 11 different range of heights.
Suppose every 3 students have the same height.
$11 \times 3 = 33$
The 34th student will be of the same height as one of the 33 students.
$33 + 1 = 34$
Therefore, the minimum class size is **34**.

210. Let 1, 2, 4, 5, 7, 9, 10, 12, 14, ... be $\{a_n\}$ and
1, 2, 2, 3, 3, 3, 4, 4, 4, 4, ... be $\{b_n\}$
We observe that $a_n + b_n = 2n$
For example,
1, 2, 4, 5, 7, 9
1, 2, 2, 3, 3, 3
$a_6 = 9, b_6 = 3$
$a_6 + b_6 = 12 = 2 \times 6$
For b_n, the last term is at 2016th, for $(1 + 2 + 3 + \cdots + 63)$
Hence $b_{2018} = 64$
$a_{2018} = 2 \times 2018 - 64$
$= \mathbf{3972}$

211. In every row, the greatest number from the previous row is removed. The remaining numbers are then written backwards.
For the 4th row, the number 7 is removed. The remaining numbers are written backwards, as shown below.

5	1	6	4

225.



$2008 = 2 \times 2 \times 2 \times 251$

We can rewrite, $2008 = 1 \times 2 \times 4 \times 251$

$1 + 2 + 4 + 251 =$ **258**

226. We know $5! = 5 \times 4 \times 3 \times 2 \times 1$
$= 120$

So, it suffices to consider up to 4!.

$1 + 2 + 6 + 24 = 33$

Ans: 3

227. We need to list down all combinations.

Suppose each of A, B, C and D represents a different club.

- There is one way for a student not to join any club.
- There are 4 ways to join only one club.
- Ways to join two clubs:
  (A, B), (A, C), (A, D), (B, C), (B, D), (C, D)
  There are 6 ways to join two clubs.
  $1 + 4 + 6 = 11$
  There are altogether 11 combinations.
  $11 + 1 = 12$

There must be at least **12 students** so that two students join the same club.

228. It is sufficient to find only the denominator.

$1 = 1$
$3 = 1 + 2$
$6 = 1 + 2 + 3$
$10 = 1 + 2 + 3 + 4$
⋮

Hence, the 50th term $1 + 2 + 3 + \ldots + 49 + 50$
$= 1275$

The 50th term is $\mathbf{\frac{1}{1275}}$.

229. From the 3rd and 4th clues, we get the following.

Thu
↑
Laura, which is then followed by Isabel and Gina.

Further, Kelly __ Jane
Heather __ __ Megan

Hence,

Tue	Wed	Thu	Fri
↑		↑	↑
Heather		Laura	Megan

Therefore,

Mon	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**	**Sun**
↑	↑	↑	↑	↑	↑	↑
Kelly	**Heather**	**Jane**	**Laura**	**Megan**	**Isabel**	**Gina**

230. Let $\overline{abcde} = n$

$\overline{2abcde} = 200000 + n$
$\overline{abcde9} = 10n + 9$

Hence, we have

$3 \times (200000 + n) = 10n + 9$
$600000 + 3n = 10n + 9$
$7n = 599991$
$n = 599991 \div 7$
$= 85713$

$\overline{abcde} =$ **85713**

231. We want to maximise the number of blue marbles. If all the marbles are blue, the maximum number of balls is 43 as $43 \times 3 = 129$ g. Using guess-and-check method, we get the following:

When $37 \times 3 = 111$ g,
$1 \times 5 = 5$ g
$2 \times 7 = 14$ g
$111 + 5 + 14 = 130$ g

Maximum number of marbles
$37 + 1 + 2 =$ **40**

232. $a = 2^{48}$
$= 2^{4 \times 12}$
$= 16^{12}$
$b = 3^{3 \times 12}$
$= 27^{12}$
$c = 5^{24}$
$= 5^{2 \times 12}$
$= 25^{12}$

Ans: Clearly, $\boldsymbol{a < c < b}$.

233. Consider the worst case scenario.

There are $200 - 70 - 20 - 32 = 78$ indecisive votes.

Since Nicholas and Patrick have more votes than Kevin, Nicholas and Patrick are likely to be the Head Prefect.

$70 + 32 + 78 = 180$
$180 \div 2 = 90$
$90 - 70 = 20$

If 20 votes go to Nicholas and $78 - 20 = 58$ votes go to Patrick, both of them will have 90 votes each.

Therefore Nicholas must have at least $20 + 1 =$ **21 more votes** to be elected as the Head Prefect.

234. The first sequence consists of all odd numbers. In the second sequence, the digit at the ones place is either "1" or "6".

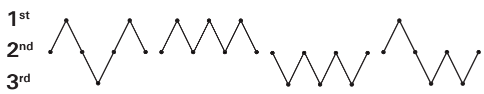
We can rewrite the two sequences as follows:

First sequence: 1, 11, 21, 31, ..., 2001
Second sequence: 1, 11, 21, 31, ..., 2001

$(2001 - 1) \div 10 = 200$
$200 + 1 = 201$

Each sequence has **201 terms**.

235. We plot Shawn's possible positions along the race.



We realise that no matter how we plot, Shawn will always come in second eventually. (Note: If overtaking occurs 7 or odd number of times, Shawn will come in the first or third.)

236.
$$\begin{array}{rl} 1^2 = 1 & \\ & +\ 4 \\ 1^2 + 2^2 = 5 & \\ & +\ 9 \\ 1^2 + 2^2 + 3^2 = 14 & \\ & +\ 16 \\ 1^2 + 2^2 + 3^2 + 4^2 = 30 & \\ \vdots & \end{array}$$

We can find a pattern of the remainders, as follows:

Term	1	5	14	30	55	91	140	204	285	385	506	650
Remainder	1	5	0	2	6	0	0	1	5	0	2	6

The remainder repeats in the pattern of 1, 5, 0, 2, 6, 0, 0.
$2009 \div 7 = 287$ R 0
Therefore, the remainder is **0**.

237. We have to capitalise on the strength of each workshop.
Workshop A:
$468 \div 12 = 39$ black pearls/day
$468 \div 18 = 26$ white pearls/day
The strength of workshop A is in producing 39 black pearls per day.
Workshop B:
$700 \div 20 = 35$ black pearls/day
$700 \div 10 = 70$ white pearls/day
The strength of workshop B is in producing 70 white pearls per day.
Workshop B produces $70 \times 21 = 1470$ white pearls in 30 days and
$35 \times 9 = 315$ black pearls in 9 days.
Workshop A produces $39 \times 30 = 1170$ black pearls in 30 days.
Total $= 315 + 1170 = 1485$ black pearls
Therefore, the maximum number of pairs of pearls produced is **1470**.

238. It is clear that $\frac{13}{24} > \frac{13}{29}$.
$\frac{14}{25} = \frac{13 + 1}{24 + 1} > \frac{13}{24}$
Hence, $\frac{14}{25} > \frac{13}{24} > \frac{13}{29}$
Next, we deduct $\frac{14}{25}$ and $\frac{24}{39}$ from a common value.
$\frac{3}{5} - \frac{14}{25} = \frac{15}{25} - \frac{14}{25} = \frac{1}{25}$
$\frac{3}{5} - \frac{24}{39} = \frac{117 - 120}{195}$ but $120 > 117$
Hence, $\frac{24}{39} > \frac{3}{5}$ and $\frac{24}{39} > \frac{14}{25}$.
Therefore, the arrangement from the greatest to the smallest is $\mathbf{\frac{24}{39}, \frac{14}{25}, \frac{13}{24}, \frac{13}{29}}$.

239. We need to construct the number of combinations. Suppose that "1" means to go for it and "0" means to give it a miss.
To visit one place: (1, 0, 0), (0, 1, 0), (0, 0, 1)
To visit two places: (1, 1, 0), (1, 0, 1), (0, 1, 1)
There are altogether 6 combinations.
$55 \div 6 = 9$ R 1
$9 + 1 = 10$
At least **10 tourists** visited the same place.

240. 18 indicates the number we are working on with descending divisors.
$20 \rightarrow 2 \times 9 = 18$ remainder 0
$22 \rightarrow 2 \times 8 = 16$ remainder 2
$24 \rightarrow 2 \times 7 = 14$ remainder 4
$30 \rightarrow 3 \times 6 = 18$ remainder 0
$33 \rightarrow 3 \times 5 = 15$ remainder 3
$42 \rightarrow 4 \times 4 = 16$ remainder 2
$m \rightarrow 6 \times 3 = 18$ remainder 0
$m =$ **60**

241. $123 + 235 + 641 = 999$
$108 + 345 + 546 = 999$
$135 + 420 + 444 = 999$
Therefore, **123, 235 and 641** are in the in the 1st group. **108, 345 and 546** are in the 2nd group. **135, 420 and 444** are in the 3rd group.

242. $525 - 331 = 194$
$913 - 525 = 388$
$1107 - 913 = 194$
388 is a multiple of 194. Hence, the difference is divisible by 194. Therefore, $n =$ **194**.
$331 \div 194 = 1$ R 137
Therefore, the remainder is **137**.

243. First, find the common multiple of 2, 3, 4, 5 and 6.
The LCM of 2, 3, 4, 5 and 6, is 60.
Hence, the possible values of A are 61, 121, 181, 241, 301, ...
However, the smallest number which is divisible by 7 is **301**.

244. $A = 1\,998\,753 \times 63\,488$
$= (1\,998\,752 + 1) \times 63\,488$
$= 1\,998\,752 \times 63\,488 + 63\,488$
$B = 1\,998\,752 \times 63\,489$
$= 1\,998\,752 \times (63\,488 + 1)$
$= 1\,998\,752 \times 63\,488 + 1\,998\,752$
Therefore, $B > A$.
B has the greater value.

245. In other words, we want to find the number of rows having the same number of students.
- If each row has a different number of students: Total number of students $(1 + 2 + ... + 30) = 465$
- If every two rows have the same number of students: Total number of students $(16 + 17 + ... + 30) \times 2 = 690$
- If every three rows have the same number of students. Total number of students $(21 + 22 + ... + 30) \times 3 = 765$
  765 is the closest to 789.
  $3 + 1 = 4$
  At least **4 rows** have the same number of seats.

246. Each term is the product of 3 and the previous term. The digits in each term are then written backwards after the 4th term.
1, 3, 9, 27, 81, 243, 729, 2187, ...
The 8^{th} term is 7812.
Therefore, n = **7812**.

247. The clues are as the follows:

Businessman	Song composer	Writer		Indonesian	Italy	Canada
×	×	✓	A	×	✓	×
×	✓	×	B	✓	×	×
✓	×	×	C	×	×	✓

B is a **song composer.**

248. Consider a block of 111 111.
111 − 111 = 0 is divisible by 7
It suffices to divide 2019 by 6.
2019 ÷ 6 = 336 R 3
$\underbrace{111\ 111}_{2019\ 1\text{'s}} = \underbrace{111\ 111}_{2016\ 1\text{'s}}000 + 111$
111 ÷ 7 = 15 R 6
Therefore, the remainder is **6**.

249. To minimise the number of rooms, we must minimise the number of rooms that can only accommodate 4 students but maximise the number of rooms accommodating a greater number of students.

4a + 7b + 11c = 46
We want to minimise the number of rooms that can only occupy 4 students. Suppose a = 0, we get b = 5 and c = 1.
5 + 1 = 6
The boys needed a minimum of 6 rooms.
4a′ + 7b′ + 11c′ = 47
Using the same method, we get a′ = 0, b′ = 2 and c′ = 3.
2 + 3 = 5
The girls needed a minimum of 5 rooms.
Therefore the minimum number of rooms was 6 + 5 = **11**.

250. Since $\frac{1}{2} < \frac{2}{3}, \frac{3}{4} < \frac{4}{5}, \ldots, \frac{49}{50} < \frac{50}{51}$,

$\frac{1}{2} \times \frac{3}{4} \times \cdots \times \frac{49}{50} < \frac{2}{3} \times \frac{4}{5} \times \cdots \times \frac{50}{51}$.

Multiply both sides by $\left(\frac{1}{2} \times \frac{3}{4} \times \cdots \times \frac{49}{50}\right)$,

$\left(\frac{1}{2}\times\frac{3}{4}\times\cdots\frac{49}{50}\right)^2 < \frac{2}{3}\times\frac{4}{5}\times\cdots\times\frac{50}{51}\times\frac{1}{2}\times\frac{3}{4}\times\cdots\times\frac{49}{50}$

$\left(\frac{1}{2} \times \frac{3}{4} \times \cdots \times \frac{49}{50}\right)^2 < \frac{1}{51} < \frac{1}{49} = \left(\frac{1}{7}\right)^2$

$A^2 < B^2$
Therefore, B > A.
B has a greater value.
(Note: This unusual problem solving technique will stretch your imagination a little further.)

251. The key lies in finding the range of different marks, and the marks that are not possibly obtained.
If all the questions are correctly answered, a total of 5 × 12 = 60 marks are awarded.
The worst case scenario is when 0 mark is obtained.
Hence the range is 0 to 60 but 1, 2, 4, 7, 57 and 59 marks are not possible.
61 − 6 = 56
There are 56 different marks in the range.
When 3 students get the same mark, there are a total of 56 × 3 = 168 students 168 + 1 = 169
Therefore, there must be at least **169 students** so that at least 4 students get the same mark.

252. 1 × 3 = 3
2 × 4 = 8
3 × 5 = **15**
4 × 6 = 24
5 × 7 = 35
6 × 8 = **48**
7 × 9 = 63
Therefore, m = **15** and n = **48**.

253. There were altogether 22 + 9 + 9 = 40 points.
Since 4 × 10 = 40, and there were 4 events, the assignment of points was either (5, 4, 1), (6, 3, 1), (5, 3, 2) or (7, 2, 1). This assumption has created a paradox as even Joan scored 6, 6, 6 and 3 from (6, 3, 1), she was still one point short of 22.
The same argument applies to (5, 4, 1) and (5, 3, 2).
As for (7, 2, 1), if Joan scored 7, 7, 7 and 1, Kelly and Linda could never score 9 points each.
Since 5 × 8 = 40, there would be 5 events and the assignment of points was either (5, 2, 1) or (4, 3, 1).
If it was (5, 2, 1):

Joan's score:	5	5	5	5	2
Kelly's score:	2	2	2	2	1
Linda's score:	1	1	1	1	5

Since Linda was the champion of the triple jump event, the last column indicates triple jump.
Therefore, **Joan** came in second in the triple jump event.

254. It is sufficient just to observe the pattern of the remainders.

Term	1	1	2	3	5	8	13	21	34	55	89
Remainder	1	1	2	3	5	0	5	5	2	7	1

Term	144	233	377	610	987	1597	2584	...
Remainder	0	1	1	2	3	5	0	...

The remainders repeat in the following pattern.

1,	1,	2,	3,	5,	0,	5,	5,	2,	7,	1,	0
↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R0

2010 ÷ 12 = 167 R 6
Therefore, the remainder is **0**.

255. If the number is $\overline{abc}$, then, $\overline{abc}$ = 25 × (a + b + c).

$$100a + 10b + c = 25a + 25b + 25c$$
$$75a = 15b + 24c$$
$$a = \frac{15b + 24c}{75}$$

When c = 5 and b = 2,

$$a = \frac{30 + 120}{75} = 2$$

The number is 252.

When c = 5 and b = 7, a = $\frac{105 + 120}{75}$ = 3

The number is 375.

375 is greater than 252.

Therefore, the greatest possible value is **375**.

256. $\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72}$

$= \left(\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5}\right) + \left(\frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9}\right)$

$= \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \ldots - \frac{1}{5}\right) + \left(\frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \ldots - \frac{1}{9}\right)$

$= \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{6} - \frac{1}{9}\right)$

$= \frac{3}{10} + \frac{1}{18}$

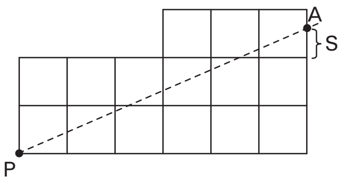
$= \frac{27 + 5}{90}$

$= \frac{32}{90}$

$= \mathbf{\frac{16}{45}}$

257. There are altogether 13 units.

Half of the land = 13 ÷ 2 = 6.5 units



Suppose that the line ends at point A, S unit of a unit is indicated.

$\frac{1}{2}(6)(2 + S) = 6.5$

$12 + 6S = 13$

$6S = 13 - 12 = 1$

$S = \frac{1}{6}$

To divide the land into half, a line has to be drawn from P to A with S = $\frac{1}{6}$ unit.

258. Let the side of the square be a.

$a^2 = 0.7a \times (a + 3)$

$= 0.7a^2 + 2.1a$

$0.3a^2 = 2.1a$

$0.3 \times a \times a = 2.1 \times a$

$a = 2.1 \div 0.3$

$= 7$ m

Area of the square = 7 × 7

= **49 m²**

259. Let the distance between the two towns be d.

$\frac{d}{60}$ is the time taken for a trip when the truck is loaded. $\frac{d}{90}$ is the time taken for a trip when the truck is empty.

$6 \times \left(\frac{d}{60} + \frac{d}{90}\right) = 12$

$\frac{5d}{180} = 2$

$5d = 2 \times 180 = 360$

d = 360 ÷ 5 = 72 km

The distance between the two towns is **72 km**.

260. Let the cost price be c.

Selling price = 1.4c × 0.9 = 1.26c

Profit = 1.26c − c = 0.26c

Actual profit earned = \$158 + \$50 = \$208

0.26c = \$208

c = \$208 ÷ 0.28 = \$800

Therefore, the cost price of the DVD player was **\$800**.

261. $\frac{135}{428} = \frac{1}{4} + \frac{28}{428}$

$\frac{200}{688} = \frac{1}{4} + \frac{28}{688}$

$\frac{268}{960} = \frac{1}{4} + \frac{28}{960}$

It is clear that $\frac{28}{960} < \frac{28}{688} < \frac{28}{428}$.

Therefore, $\frac{135}{428} > \frac{200}{688} > \frac{268}{960}$.

The largest fractions to the smallest fractions are $\mathbf{\frac{135}{428}, \frac{200}{688}, \frac{268}{960}}$.

262. Combined rate = $\frac{1}{9} + \frac{1}{15} = \frac{8}{45}$

After 8 days,

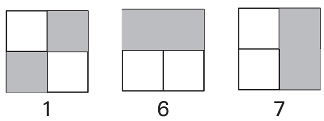
$\frac{8}{45} \times (8 \div 2) = \frac{8}{45} \times 4 = \frac{32}{45}$ of the crops are harvested.

Fraction of crops left = $\frac{4}{5} - \frac{32}{45} = \frac{4}{45}$

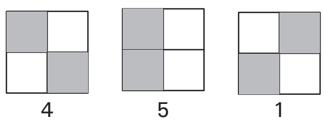
On the 9th day, Mark needs $\frac{4}{45} \div \frac{1}{9} = \frac{4}{5}$ day

It takes $\mathbf{8\frac{4}{5}}$ days in all.

263. Observe the pattern carefully. Since 167 represents the first floor,



Hence, using the same method, the 3 frames represent 451.



Hence, we get the following:

1st floor → 167

2nd floor → 205

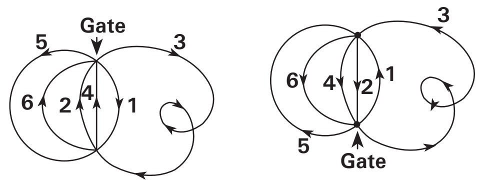
3rd floor → 451

4th floor → **983**

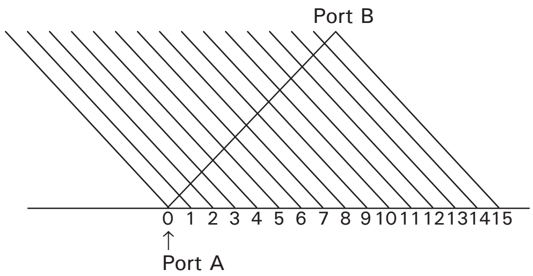
5th floor → 640

6th floor → 385

264. Two possible locations of the gate.



265. This can be solved by drawing the figure below.



7 days + 8 days = 15 days

It will meet a cargo at Port A and another at Port B.

It will also meet 14 cargoes along the way

Therefore, it will meet altogether **16 cargoes.**

266. Let t be the time for the candles to bum.

Fraction of candle A burnt $= \frac{t}{60 \times 4} = \frac{t}{240}$

Fraction of candle B burnt $= \frac{t}{60 \times 3} = \frac{t}{180}$

Fraction of candle A remaining $= \left(1 - \frac{t}{240}\right)$.

Fraction of candle B remaining $= \left(1 - \frac{t}{180}\right)$.

The ratio of the remaining candles is 2 : 1.

$\left(1 - \frac{t}{240}\right) : \left(1 - \frac{t}{180}\right) = 2 : 1$

$1 - \frac{t}{240} = 2 \times (1 - \frac{t}{180}) = 2 - \frac{t}{90}$

$\frac{t}{90} - \frac{t}{240} = 2 - 1$

$\frac{8t - 3t}{720} = 1$

$5t = 720$

$t = 720 \div 5$

$= 144$ minutes

$= 144 \div 60$

$= 2$ h 24 min

It takes 2 hours 24 minutes for the candles to burn.

The candle must be lit at **11.36 am.**

267. Area of lawn $\rightarrow \frac{7}{9}$

Area of pond $\rightarrow 1 - \frac{7}{9} = \frac{2}{9}$

If P is the area of the pond, area of the lawn is

$\frac{7}{2}P = 3.5P$.

Area of garden $\rightarrow \frac{3}{4}$

Area of pond $\rightarrow 1 - \frac{3}{4} = \frac{1}{4}$

Hence, the area of the garden is 3P.

$3.5P - 3P = 0.5P$

The difference in the area of the lawn and that of the garden is 200 m^2.

$0.5P \rightarrow 200$

$P \rightarrow 200 \times 2$

$= 400$ m^2

Area of the pond = **400 m^2**

268. Method 1:

$(1 + 2) \times 2 = 6$

$(6 + 2) \times 2 = 16$

$(16 + 6) \times 2 = 44$

⋮

We find the pattern of the remainder when each term is divided by 9.

Term	1	2	6	16	44	120	328	896	2448	6688
Remainder	1	2	6	7	8	3	4	5	0	1
	↑ R1	↑ R2	↑ R3	↑ R4	↑ R5	↑ R6	↑ R7	↑ R8	↑ R0	

The pattern repeats in the pattern 1, 2, 6, 7, 8, 3, 4, 5 and 0.

$2011 \div 9 = 223$ R 4

Therefore, the remainder is 7.

Method 2:

From the third term onwards, it is sufficient to sum up the remainders of the previous two terms and multiply it by 2 before dividing it by 9.

$(1 + 2) \times 2 \div 9 = 0$ R 6

$(6 + 2) \times 2 \div 9 = 1$ R 7

$(7 + 6) \times 2 \div 9 = 2$ R 8

$(8 + 7) \times 2 \div 9 = 3$ R 3

⋮

Hence we have 1, 2, 6, 7, 8, 3, 4, 5 and 0.

$2011 \div 9 = 223$ R 4

Therefore, the remainder is **7**.

269. To minimise the shipment cost, Hong Kong branch must maximise the number of books delivered to Korea and Singapore branch must maximise the number of books delivered to the Phillipines.

The costs of shipment are summarised in the table below where 1 unit → 200 copies.

Branch	Korea	Phillipines
Hong Kong	\$300 × 10 units	\$500 × 2 units
Singapore		\$400 × 6 units

$\$300 \times 10 + \$500 \times 2 + \$400 \times 6$

$= \$3000 + \$1000 + \$2400$

$= \$6400$

Least cost to deliver the books = **\$6400**.

270. It helps when we know the number of times the cards have been distributed.

Total scores $= 22 + 20 + 18 = 60$

Average score of each player $= 60 \div 3 = 20$

This concludes that the cards are distributed, at most, 3 times.

Possible values of the numbers on the cards:

(8, 8, 4), (8, 7, 5)

(9, 6, 5), (9, 7, 4)

(8, 8, 4) cannot make a total of 22, likewise for (8, 7, 5).

As for (9, 6, 5):

$9 + 6 + 5 = 20$

$6 + 6 + 5 = 17$

$9 + 9 + 5 = 24$

We also cannot get a possible combination from (9, 6, 5).

As for (9, 7, 4):

$9 + 9 + 4 = 22$

$7 + 4 + 9 = 20$

$4 + 7 + 7 = 18$

Therefore, the numbers on the cards are **9**, **7** and **4**.